ACTIVITY 2

Old MacDonald's Pigpen

Math Objectives:

- Graph scatter plots
- Graph and analyze quadratic functions
- Calculate the maximum value of a parabola

Materials:

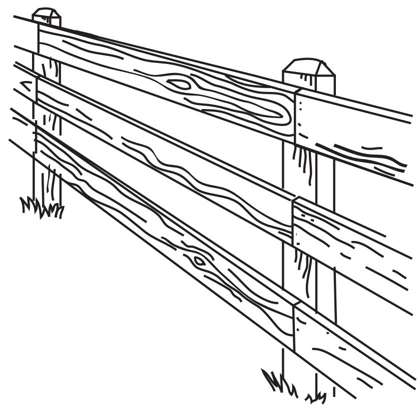
- TI-83/TI-84 Plus Family

OVERVIEW

Old MacDonald has 40 meters of fencing to make a rectangular pen for his pigs. If he wants to give the little pigs as much room as possible, what should be the dimensions of the pen?

This simple version of a standard maximum value problem provides the students with the opportunity to use extra features on the calculator. These calculator functions will save time and ensure accuracy.

This problem is easily adapted for pre-algebra, algebra, or even pre-calculus. Pre-algebra students have the ability to comprehend the idea that the area varies as the width varies. They can then grasp that it is likely there is one rectangular pigpen with the greatest area and the calculator can identify that particular rectangle for them. Students who have already studied parabolas and their quadratic equations can be encouraged to match the mathematical theory to the results found by the calculator.



DATA COLLECTION

1. Allow students to work in groups to fill in the chart on their worksheet. Then guide them in a discussion toward finding a general formula for calculating the length. Ask the students, "If 2 of the lengths plus 2 of the widths is forty, what would be the sum of one of the lengths plus one of the widths?" Lead the students from $L + W = 20$ to the equation $L = 20 - W$.

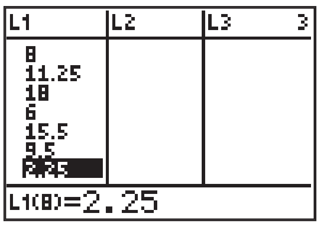


Figure 1

2. Have the students enter the widths of the rectangle in **L1**. **See Figure 1.**

★ **NOTE** *For help with entering data into lists, see Appendix B.*

3. Rather than typing in all the lengths from their charts, show the students how the calculator can do the work for them. Define **L2** with the formula for calculating the length arrived at in the discussion. Use the up arrow key and move the cursor to highlight **L2**. Type **"20–L1"**. Press [ALPHA] [+] to access the quotation marks and [2nd] [1] to enter **L1** into the equation. **See Figure 2.**

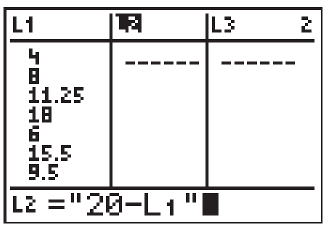


Figure 2

NOTE Enclosing the formula in quotation marks will automatically recalculate **L2** whenever you make a change to **L1**.

4. Whenever the name of a list is highlighted, the command entered will be applied to the entire list. What is typed will appear at the bottom of the screen. When [ENTER] is pressed, the highlighted list is generated. **See Figure 3.**

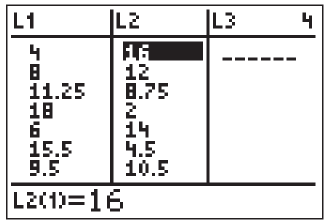


Figure 3

5. Use the calculator to also find the perimeter and area. With the cursor highlighting **L3**, type "**2L1+2L2**". The keystrokes are [ALPHA] [+] [2] [2nd] [1] [+] [2] [2nd] [2] [ALPHA] [+]. **See Figure 4.**

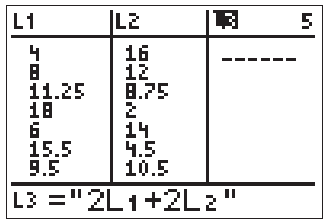


Figure 4

6. Press [ENTER]. Because the perimeter is 40 for each pair of dimensions, this confirms that each rectangle is using all 40 meters of fencing. **See Figure 5.**

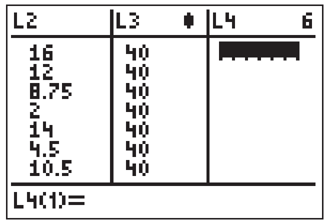


Figure 5

7. Next, highlight **L4** and type "**L1L2**". The keystrokes are [ALPHA] [+] [2nd] [1] [2nd] [2] [ALPHA] [+]. **See Figure 6.**

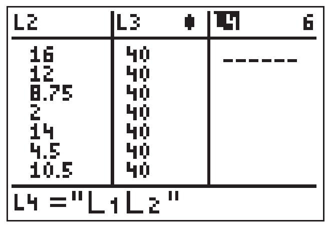


Figure 6

8. Press [ENTER] to see **L4** fill in with the areas of the rectangles generated by the area formula. **See Figure 7.**

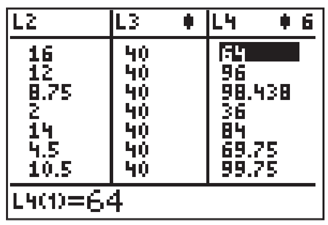


Figure 7

9. The **L3** list does not need to be displayed for the next steps. Only three lists can be viewed at a time and the lists **L1**, **L2**, and **L4** need to be in view. The order of the lists can be changed so that the lists that are needed will appear on the screen. To do this, put the cursor on **L3** and press the [DEL] key. **See Figure 8.**

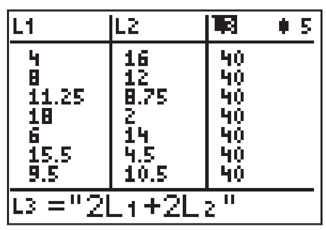


Figure 8

10. This procedure will delete the list from being shown in the display but it does not erase the values in the list. The three lists that are needed for the rest of this investigation can now be viewed on the calculator screen. **See Figure 9.**

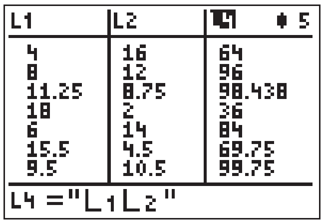


Figure 9

NOTE *For help with adjusting the display of lists, see Appendix B.*

11. To more readily identify a pattern, it would be helpful to have the widths listed in order, from smallest to largest.

12. Let the calculator sort the list. Press the STAT key and select **2:SortA(** from the menu. This function will sort the list in ascending order. **See Figure 10.**

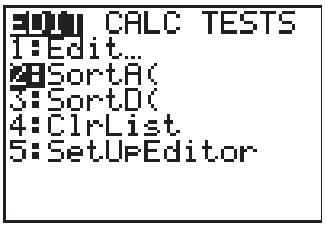


Figure 10

13. This entry moves you to the home screen. Type 2nd 1 to enter **L1** and then press). Press ENTER to execute the command. The calculator has now sorted the lists. **See Figure 11.**

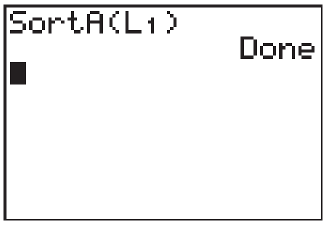


Figure 11

NOTE If **L1** is entered, the calculator will arrange the numbers in **L1** in order from least to greatest. This is where the importance of the quotation marks is demonstrated. Unless the other lists were built with quotation marks, the calculator will leave the numbers in the other lists unsorted. This would result in the widths entered not matching up with the correct lengths, perimeters, or areas. Because the numbers in the other lists are related to the numbers in the first list, the entire row has to be carried along with the lead entry from **L1** when it is sorted. Because **L2** and **L4** were created with quotation marks, their entries will follow the entries in **L1**. If lists are built without quotation marks, then type **SortA(L1**, **L2**, **L4)** to sort the rows so that the values that relate remain together.

14. Go to STAT, **1:Edit…** Notice that the data has been sorted and that **L1** is listed in ascending order. Each width’s corresponding length and area have remained in line with it. **See Figure 12.**

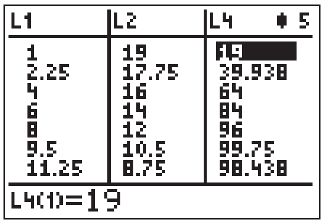


Figure 12

DATA ANALYSIS

1. Now press 2nd Y= to access [STAT PLOT]. Highlight **On** and press ENTER. Select the scatter plot icon to create a scatter plot. Use **L1** for the **Xlist** and **L4** for the **Ylist.** These entries will allow the area, **L4**, to be graphed as a function of the width of the rectangles, **L1**. This is a good time to review vocabulary such as “**Y** is a function of **X”** which leads to “the area is a function of the widths.” Help students identify the width as the independent variable and the area as the dependent variable. **See Figure 13.**

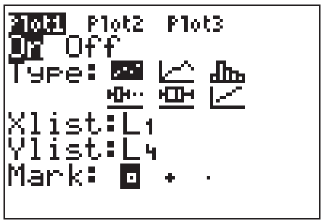


Figure 13

NOTE *For help setting up a scatter plot, see Appendix F.*

2. Press ZOOM. Scroll down to highlight **9:ZoomStat** and press ENTER or press 9 instead. The points plotted should be in view. Press TRACE and scroll right and left to see the **X**- and **Y**-values of the data points. Ask students to name the shape of the graph if the points were connected. Lead them to realize it looks like a parabola and tell them that the calculator can find the equation of the parabola for them. **See Figure 14.**

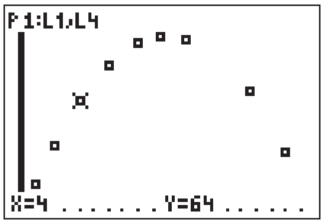


Figure 14

3. Find the regression equation and paste it in **Y1**. To do this, press [STAT] and scroll over to **CALC** and down to **5:QuadReg**. Press [ENTER]. When **QuadReg** appears on the home screen type **L1**, **L4**, **Y1** after it. This tells the calculator which lists to use for the **X**- and **Y**-values and where to paste the regression equation. The keystroke sequence is as follows: [2nd] [1] [,] [2nd] [4] [,] [VARS] [▶]. Choose **1:Function** and then **1:Y1**. ([2nd] [1] is used to access **L1** and [2nd] [4] will access **L4**.) **See Figure 15.**

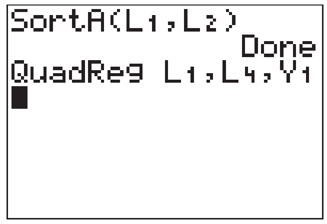


Figure 15

★ **NOTE** *For more help with finding regression equations, see Appendix G.*

4. Press [ENTER]. The regression equation will be displayed on the home screen. **See Figure 16.**

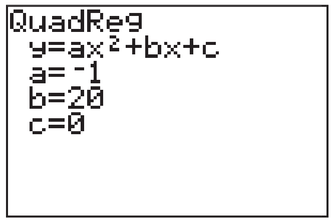


Figure 16

5. Go to the [Y=] window. Notice that the coefficients are very long decimal numbers. Although the **a** value on the home screen was –1 and the **b**-value was 20, here the **a-** and **b-**values have rounding errors. This is a great time for a discussion about the fact that the calculator is a tool and it is only as good as the person using it. Students need to recognize that –.99999999 is equivalent to –1 and 19.999999999 is equivalent to 20. **See Figure 17.**

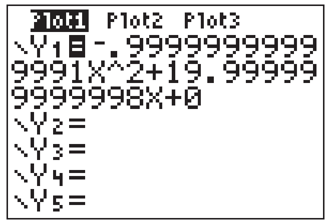


Figure 17

6. Press [GRAPH] and watch the calculator connect the points that were already plotted. Examine how closely the regression equation fits the points. **See Figure 18.**

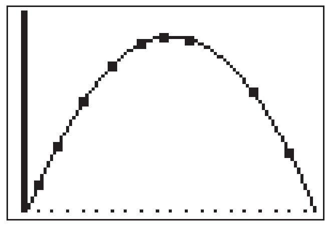


Figure 18

7. Next, find the maximum area. Press [2nd] [TRACE] to access the [CALC] menu and scroll down to **4:maximum. See Figure 19.**

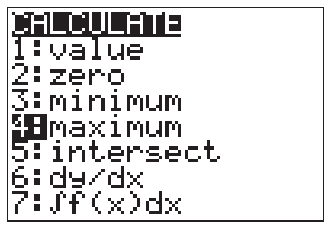


Figure 19

8. Press [ENTER]. This takes you back to the graph. Follow the onscreen directions to identify the maximum area. When asked for the **Left Bound?**, move the left arrow key until the cursor is clearly to the left of the vertex and press [ENTER]. That choice is marked and the onscreen question changes. **See Figure 20.**

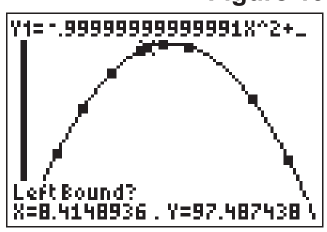


Figure 20

9. When asked for the **Right Bound?**, move the right arrow key until the cursor is clearly to the right of the vertex and press [ENTER]. Again, the choice is marked and the onscreen question changes. **See Figure 21.**

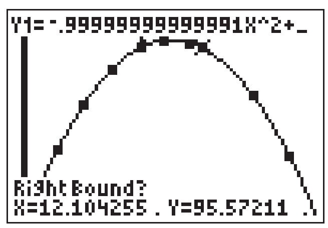


Figure 21

10. When asked for a **Guess?**, move the cursor close to the vertex point and press ENTER. **See Figure 22.**

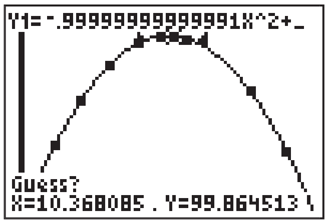


Figure 22

11. The maximum point of the graph is ***x*** = 10.000004 and ***y*** = 100. Since the ***x***-term corresponds to the width, 10 meters is the maximum width needed to form a rectangle that contains the maximum area. The ***y***-term corresponds to the area, so $100m^2$ is the maximum area of the rectangular pigpen. (answers may vary) **See Figure 23.**

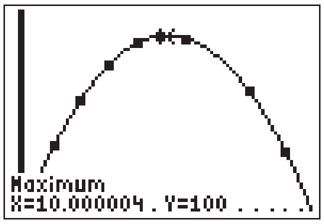


Figure 23

DISCUSSION NOTES

Guide your students in a discussion that will lead them to an understanding of which variables stand for which values in this problem. You should be able to count on students at this level knowing that a rectangle's Area = length x width. Help them see this problem can be that simple.

Substituting x for the width, and $20 – x$ for the length, show them that the Area=$(20 – x)x$ or $A(x)=20x – x^2$. In standard form, the equation becomes $A(x) = –x^2 + 20x$. Match this equation to the standard quadratic equation, $y = ax^2 + bx + c$, with a = –1, b = 20, and c = 0 as the coefficients. These are precise constants that are not rounded. They help to conclude that those long decimal numbers in the Y= screen were the result of rounding errors.

Pre-algebra students will need to depend on the calculator to identify the maximum values but algebra students who have studied quadratics can identify the **X**-value of the vertex as being found at –b/2a. For the values in this problem, the vertex would be –(20)/2(–1) = 10. This is the width of the rectangle with the largest area. The maximum area would be 10(20 – 10) = 100. The worksheet is designed to help lead students through this same reasoning.

WORKSHEET ANSWERS

1–10. See chart

11. $A(x) = x\,(20 – x)$

12. quadratic, parabola

13. vertex

14. width

15. area

16. width

17. maximum area

Rect #	Width	Length	Per	Area
1.	4 m	16	40	64
2.	8 m	12	40	96
3.	$11\frac{1}{4}$ m	8.75	40	98.438
4.	18 m	2	40	36
5.	6 m	14	40	84
6.	$15\frac{1}{2}$ m	4.5	40	69.75
7.	$9\frac{1}{2}$ m	10.5	40-	99.75
8.	$2\frac{1}{4}$ m	17.75	40	39.983
9.	1m	19	40	19
10.	x	$20 - x$	40	$x(20 - x)$

Activity 2

Name: ______________________________

Old MacDonald's Pigpen

Math Objectives:

- Graph scatter plots
- Graph and analyze quadratic functions
- Calculate the maximum value of a parabola

Materials:

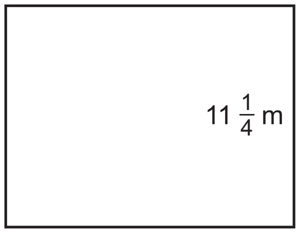
- TI-83/TI-84 Plus Family

Old MacDonald has 40 meters of fencing to make a rectangular pen for his pigs. If he wants to give the little pigs as much room as possible, what should be the dimensions of the pen? He started to figure this out by drawing a few sample rectangles below. Mark the sides of the following rectangles with the lengths that would ensure that each rectangle has a perimeter of 40 meters. Fill in the chart on the next page. Make sure the perimeter is always 40 meters. Find the area of each rectangle.

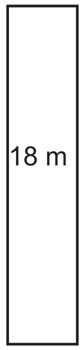
1.

2.

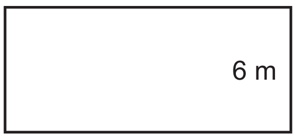
3.



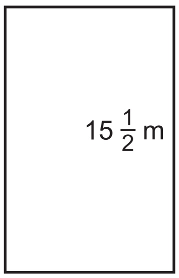
4.



5.



6.



7.

8.

9.

10.

Rectangle #	Width	Length	Perimeter	Area
1.	4 m			
2.	8 m			
3.	$11\frac{1}{4}$ m			
4.	18 m			
5.	6 m			
6.	$15\frac{1}{2}$ m			
7.	$9\frac{1}{2}$ m			
8.	$2\frac{1}{4}$ m			
9.	1m			
10.	x			

11. The chart makes it plain that the area of the rectangles is a function of the length of the sides. Use **X** to denote the width and write an equation that relates the area to **X**. This is no more difficult than saying Area = length x width. A(x)=____________________.

12. This is a ____________________ equation. Its graph will be a ____________________.

13. To find the maximum area, find the ____________________ of the graph.

14. The **X**–value stands for the ____________________ of the rectangles.

15. The **Y**–value stands for the ____________________ of the rectangles.

16. The **X**–value of the vertex tells us the ________________ of the rectangle with the maximum area.

17. The **Y**–value of the vertex tells us the value of the ____________________.

NOTES

