EXPLORATIONS

Activity 12

Objective

- To investigate exponential growth in powers of 2

Materials

- TI-73 calculator
- Student Worksheet

A Penny Saved is a Penny Earned

In this activity you will:

- calculate the amount of money an individual would receive using the doubling plan described below
- investigate patterns in powers of 2

You will need to know this math vocabulary:

- exponent
- power
- base
- factor
- exponential form
- scatterplot

Introduction

You may have heard of this age-old problem: A very wealthy family offered their son these choices if he agreed to do his daily chores for an entire month. These were his choices:

- *Plan A*: They would give him 1 penny the first day, 2 pennies the second day, 4 pennies the third day, 8 pennies the fourth day and continue doubling the previous day allowance until the end of the 31-day month.
- *Plan B*: Pay him a fixed amount of $1,000,000.

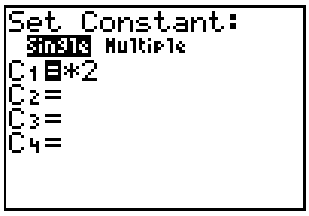
Which offer should he accept: Plan A or B?

Problem

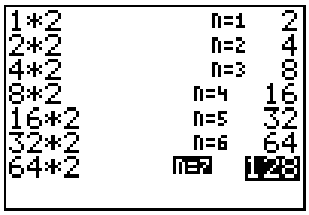
In this activity, you will determine which offer will be more valuable by finding the difference in the two offers. You will investigate the graph of this model and some patterns involved.

Activity

1. To constantly double the previous amount, you can use the constant function of the calculator. Press [2nd] [SET] and use the setup shown at the right.

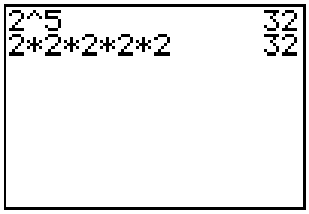


2. Press [2nd] [QUIT] [CLEAR] to go to the Home screen and clear it. Type [1] [CONST] and continue pressing [CONST] as the counter counts the number of times you multiply by 2.



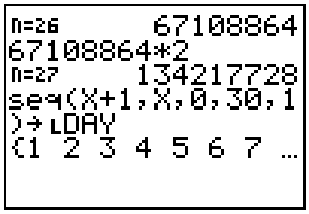
✎ Complete Table 1 and answer questions 1 through 4 on the Student Worksheet.

3. You may be familiar with expressing repeated multiplication by the same factor in exponential form. Recall that 2^5 is said to be in **exponential form** where 2 is the **base** and 5 is the **exponent.** The 5 tells how many times 2 is taken as a **factor**. Use your calculator to find the value of 2^5. Press [2] [^] [5] [ENTER].



✎ Complete Table 2 on the Student Worksheet.

4. Use the list editor to investigate Plan A. Name a list **DAY** to represent the day number of the 31-day month.



a. To save time in entering the numbers 1-31 in this list, you can use the **sequence** command. Press [2nd] [STAT] [▸] **7:seq(** and then use the arguments shown in the screen at the right. Store this sequence to the list named **DAY**.

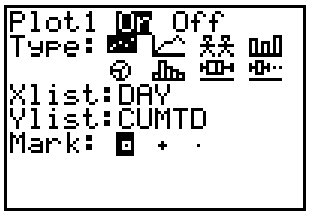
b. Go to the list named **DAY** and you should have the numbers 1-31 entered as elements of this list. To the right of **DAY**, name two new lists **PPAID** (pennies paid), and **DPAID** (dollars paid). Instead of typing in each element you can write formulas to generate the lists. Study Table 2 and think of the list name **DAY** as variable **X** in the bottom row of Table 2. If you replaced **DAY** with **X**, what would the formula be to generate the **PPAID** (pennies paid) list? Discuss it with your group. Try it to see if it generates the correct values.

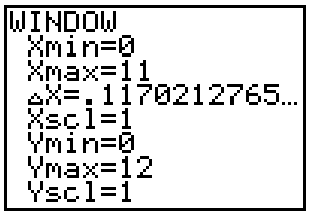
✎ Answer questions 5 and 6 on the Student Worksheet.

c. Now you will use the calculator to compute the cumulative total amounts after each day. Name 2 more new lists **CUMTP** (cumulative total in pennies) and **CUMTD** (cumulative total in dollars).

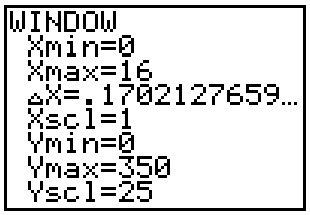
✎ Answer questions 7 through 11 on the Student Worksheet.

5. Define Plot 1 as a **scatterplot** by pressing [2nd] [PLOT] [ENTER] and define as shown in the illustration. Set up the following window to look at the first 10 days. Press [GRAPH] [TRACE]. Then adjust it to look at the first 15 days. Press [GRAPH] [TRACE]. Finally, adjust it to look at the first 30 days. Press [GRAPH] [TRACE].



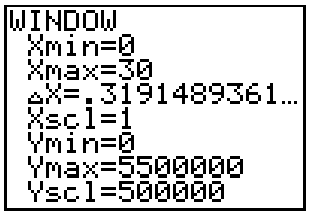


a. To get a better idea of the shape of the graph, you can enter an equation into the [Y=] editor. If **X** = the day number and **Y** = the cumulative total amount paid in dollars, write an equation to describe this. Enter this equation into the [Y=] editor. **Y1**= (2^X - 1)/ 100. Press [GRAPH].



✎ Answer question 12 on the Student Worksheet.

b. Finally, create a graph of Plan B. Since it is a fixed $1,000,000, the equation is Y=1000000. Enter this equation in **Y2** . Press [GRAPH].



✎ Answer question 13 on the Student Worksheet.

c. Create graphs to investigate both plans. Press [2nd] [PLOT] **4:PlotsOff** [ENTER] to turn the plots off.

Student Worksheet

Name _____

Date _____

Activity 12

A Penny Saved is a Penny Earned

Record your results on the table below. Then answer the questions about the activity.

Table 1

	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 10	Day 20
N=									
amount earned in pennies	1	2	4						

1. How much will be earned in cents on the seventh day?

_____

2. What is the **total** amount earned in cents and in dollars after 7 days?

_____

3. How does the counter value (N) compare to the day number?

_____

4. $1,000,000 = _____ pennies. Continue using constant key to determine the single day where Plan A will pay more than $1,000,000 in pennies. On what day will this occur? Record the amount in pennies and in dollars.

_____

_____

Table 2

Day	Pennies Paid	Power of 2	Cumulative total
1	1		1
2	2		1+2=3
3	4		1+2+4=7
4	8	2^3	7+8=15
5	16		31
6			
7			
10			
X			

5. If you replaced DAY with **X**, what would the formula be to generate the **PPAID** (pennies paid) list?

_____

_____

6. Write a formula to convert pennies paid to dollars paid in the **DPAID** list.

_____

7. Write a formula to express the cumulative total paid in pennies.

_____

8. Write a formula to express the cumulative total paid in dollars.

_____

9. Scroll down the **CUMTD** and find the total that would be paid at the end of the 31-day month. A second way to answer this question would be to sum the list named **DPAID**. ([2nd] [STAT] ▶ ▶ **7:sum(DPAID** [)] [ENTER]**)**

_____

10. Scroll the list and find the day that plan A would exceed plan B.

_____

11. How much more money would be paid on Plan A than Plan B?

_____

_____

12. Describe the graph of Plan A.

_____

_____

_____

13. Describe the graph of Plan B.

_____

_____

_____

Teacher Notes

Activity 12

A Penny Saved is a Penny Earned

Math Strand

- Algebraic reasoning
- Number sense
- Statistics

Materials

- TI-73 calculators (one per pair or one for each student)
- Student Worksheets (page 114)

Students will explore exponential growth using powers of 2 and discover patterns in their sums.

Vocabulary

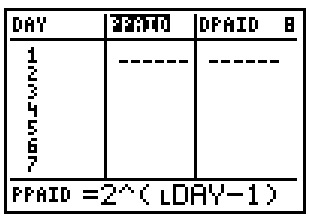
exponent	In a^b, **b** is the exponent and tells how many times **a** is used as a factor.
power	the exponent
base	In a^b, **a** is the base and is used as a factor **b** times.
factor	a number or variable to be multiplied
exponential form	written using exponents
scatterplot	a graph that shows the general relationship between two sets of data

Classroom Management

Students should work in teams of 2 to 4.

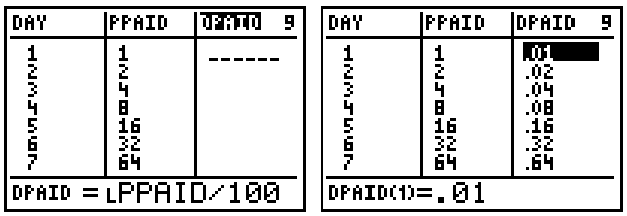
Activity

4. If students have limited background in using lists, you may need to lead this activity. For information about accessing lists, naming lists, and using formulas in lists, see Appendix A, B, and C respectively.

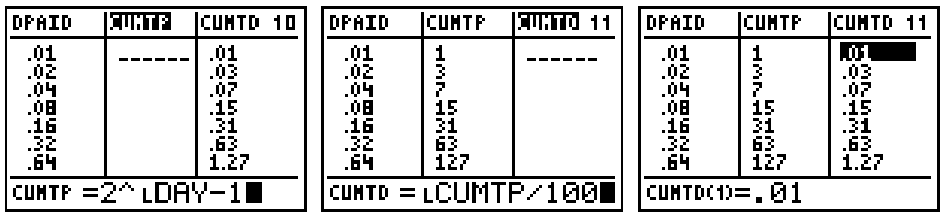


a. If students are not familiar with using formulas in lists, you may want to lead the part of the activity where they are developing a formula to generate the pennies paid. Ask the students what they notice about the **DAY** number and the amount paid on that given day. The amount paid in pennies is a power of 2. *It is 2 to the power of the quantity of the day number minus one.*

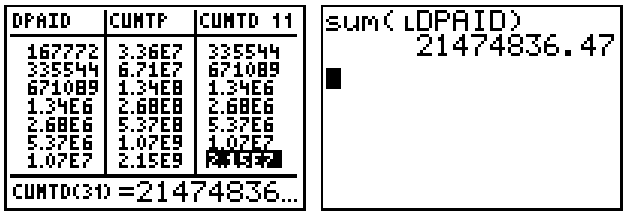
Ask the students how they convert from pennies (cents) to dollars. *Take the PPAID list and divide by 100.*



b. Next, the students will derive a formula to generate a cumulative total amount paid after each day. Note the pattern in Table 2. The cumulative sum is 1 less than a power of 2. *It is 2 to the power of the day number minus one.*



c. Using the list to answer question 9 requires an understanding of scientific notation.



Answers to the Student Worksheet

	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 10	Day 20
N=		1	2	3	4	5	6	9	19
amount earned in pennies	1	2	4	8	16	32	64	512	524288

1. 64

2. 127, $1.27

3. It is one less. N-1 = Day

4. 100,000,000 pennies, Day 26; 134,217,728 pennies = $1,342,177.28

Table 2

Day	Pennies Paid	Power of 2	Cumulative total
1	1	2^0	1
2	2	2^1	1+2=3
3	4	2^2	1+2+4=7
4	8	2^3	7+8=15
5	16	2^4	31
6	32	2^5	63
7	64	2^6	127
10	512	2^9	255
X	2^(X-1)	2^(X-1)	2^X - 1

5. **PPAID** = 2^(**DAY** - 1)

6. **DPAID** = **PPAID**/100

7. **CUMTP** = 2^**DAY** -1

8. **CUMTD** = **CUMTP**/100

9. About 2.15×10^7 or exactly $21,474,836.47.

10. After 27 days, Plan A would exceed Plan B.

11. Plan A would pay $20,474,836.47 more than Plan B.

12. Answers will vary. It is a curve that rapidly increases (increases exponentially).

13. Answers will vary. It is a straight horizontal line that remains constant at $1,000,000.

Going Further

Look at other exponential growth or decay models using powers other than 2, such as world population.

Students can continuously fold a piece of paper in half.

- Analyze the layers-exponential growth.
- Analyze the regions-exponential growth.
- Analyze the areas of the regions - exponential decay.