Factoring Special Cases

ID: 9605

Time required
30 minutes

Activity Overview

In this activity, students explore geometric proofs for two factoring rules: $a^2 + 2ab + b^2 = (a + b)^2$ and $x^2 - a^2 = (x - a)(x + a)$. Given a set of figures whose combined areas represent the left-hand expression, they manipulate the figures to create rectangles whose areas are equal to the right-hand expression.

Topic: Quadratic Functions & Equations

- *Express a trinomial square of the form $a^2 + 2ab + b^2$ as the binomial squared $(a + b)^2$.*
- *Express a difference of squares of the form $x^2 – a^2$ as $(x – a)(x + a)$ and display as a difference of areas.*

Teacher Preparation and Notes

- *This activity is appropriate for students in Algebra 1. Prior to beginning this activity, students should be familiar with factoring quadratic expressions. The activity should be followed by additional practice applying the rules discussed.*
- *This activity requires students to drag, rotate, and hide objects, as well as label them with the Text tool. If students are not familiar with these TI-Nspire functionalities, extra time should be taken to explain them.*
- *Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.*
- ***To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "9605" in the keyword search box.***

Associated Materials

- *FactoringSpecialCases_Student.doc*
- *FactoringSpecialCases.tns*
- *FactoringSpecialCases_Soln.tns*

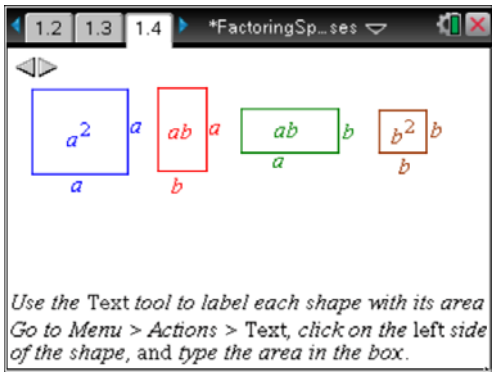
Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Polynomials, Factors, Roots and Zeros (TI-Nspire Technology) — 16044*
- *Factoring Trinomials (TI-Nspire Technology) — 16046*
- *More Difficult Factoring (TI-84 Plus family) — 5074*

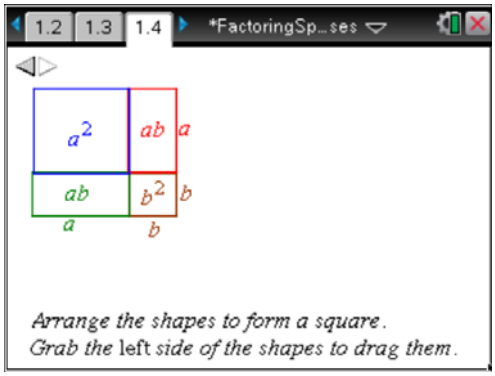
Problem 1 – Factoring a perfect-square trinomial

Page 1.2 introduces the activity and page 1.3 presents the rule for factoring a perfect-square trinomial.



On page 1.4, students are given four figures: a square with side length *a*, two rectangles with side lengths *a* and *b*, and a square with side length *b*. Students should calculate the area of each of these and label each shape with its area.

Then, students will rearrange the shapes to form a square. For some students, the arrangement may be obvious; others may have more difficultly.



Page 1.5 discusses the area of the square that students formed. Help students to see that each side of the square is made up of two pieces, one with length *a* and one with length *b*, so the length of each side is the sum of these, $a + b$. You may choose to have students choose two different numbers to substitute for *a* and *b* to see that this is true.

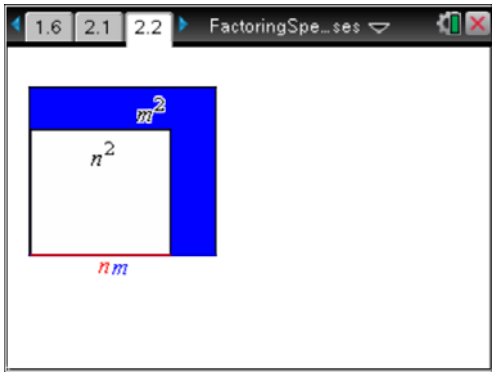
Once they are convinced of the dimensions of the square, students should use them to calculate its area. Page 1.6 restates the conclusion of the proof.

TI-Nspire Navigator Opportunity: *Screen Capture*

See Note 1 at the end of this lesson.

Problem 2 – Factoring a difference of squares

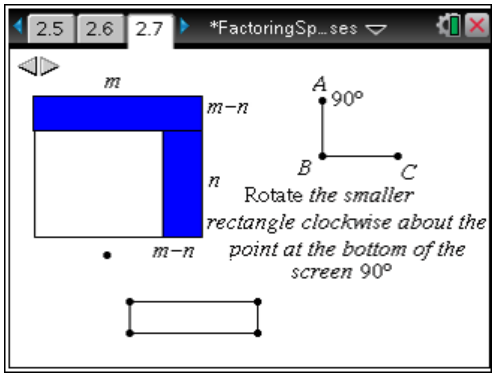
In this problem, students perform a slightly more complicated manipulation to prove the rule for factoring a difference of squares. On page 2.2, students are given a square with side length *m* and a square with side length *n*. They should label each square with its area.



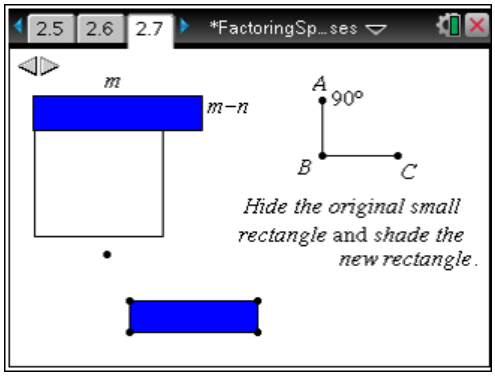
To represent $m^2 - n^2$, students should "subtract" the area of the smaller square from that of the larger by first changing the color of the smaller square to white (to represent $-n^2$).

Next, students should drag the white square over top of the larger shaded square so that their corners align. This is analogous to "cutting out" and removing the smaller square from the larger. The shaded L-shaped area that remains visible is equal to $m^2 - n^2$.

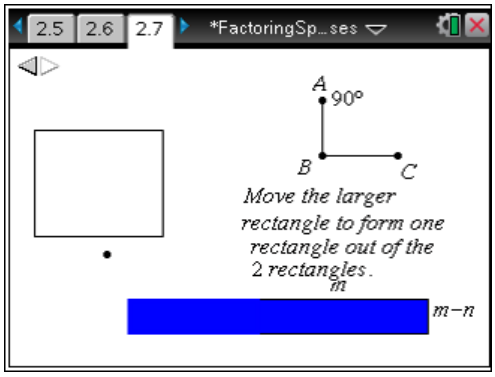
On page 2.7, students are shown another copy of the diagram along with some tools to help them transform the shaded L-shape into a long rectangle with the same area. The L-shape has already been divided into two rectangles for them.



They should rotate the smaller of the rectangles clockwise 90° by selecting **MENU > Transformation > Rotation**, clicking the point about which to rotate the rectangle (the point at the bottom of the screen), clicking the rectangle, then marking the angle that they want to rotate the rectangle by clicking *A*, then *B*, then *C*. Do NOT click the 90° measurement.



Then next step is to hide the original smaller rectangle (the one that they just rotated). Now there are two rectangles whose combined area is equal to the area of the original L-shape. They should also hide the vertices of the new rectangle. (To hide an object, move the cursor over it, press ctrl + menu, then choose **Hide/Show**.)



Finally, they should drag the ***larger*** rectangle (since the smaller rectangle cannot be moved) along side the smaller one to form one long rectangle.

Page 2.8 asks students to find the dimensions and area of this rectangle and page 2.9 restates the rule that they have proved.

TI-Nspire Navigator Opportunities

Note 1

Problem 1, *Screen Capture*

Do a *Screen Capture* on page 1.4 to monitor student progress on dragging the rectangles to form a square.