The Derivative of Logs

DerivativeOfLogs.tns

Name _____

Class _____

Problem 1 – Finding the derivative of the y = ln x

On page 1.4, you will see a series of open circles on the x-axis as you manipulate the slider, as well as a tangent drawn to the graph of y = ln x. The slope of the tangent is shown at the top of the screen.

Each open circle will appear as you manipulate the slider at the bottom of the screen. Your goal is to move the tangent line such that the point of tangency is located directly above or below each of the 9 points (open circles), starting with the leftmost point.

The next step is to move the corresponding point on the x-axis vertically so that it's y-coordinate is approximately equal to the slope of the tangent. In the diagram shown, the point will be moved up so that its y-coordinate is approximately 1.9.
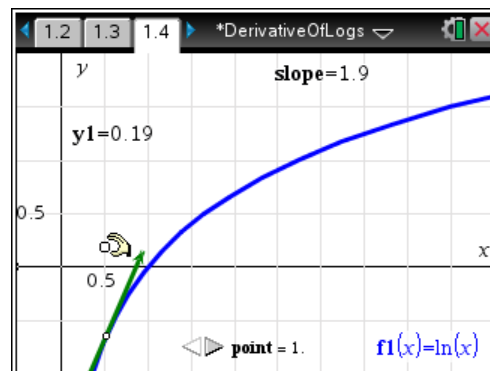
After placing all points, use the **Hide/Show** tool to reveal point p. Click once on point p. This point represents the derivative of y = ln x at the point of tangency. Press ⎡esc⎤ to exit the **Hide/Show** tool.

Now select the **Geometry Trace** tool and click on point p. Then, drag the tangent line. This represents the approximate graph of the derivative.

- What function do you think is represented by this scatterplot?

To confirm your conjecture, press ⎡ctrl⎤＋ ⎡G⎤ to show the entry line and enter your guess for the derivative of the function y = ln x in **f2**(x). If you guessed correctly, your graph will pass closely through the trace.

Advance to page 1.6.

Confirm your function by setting up and evaluating a limit expression, based on the definition of the derivative. Use the **Limit** command from the Calculus menu or select the limit template from the Math Templates palette.

- What is the limit expression? What is the value of the limit?

Next, use the **Derivative** command as a final check of your result.

- What is the derivative of y = ln x?

Problem 2 – Finding the derivative of any logarithmic function $y = \log_b x$

Use the change-of-base formula to express $\log_b x$ in terms of natural logarithms. Take the derivative of this expression (knowing the derivative of ln x from Problem 1) to find the derivative of $y = \log_b x$. Show your work below.

Confirm your result by using the **derivative** command to find the derivative of $y = \log_b x$. You may find that this result does not appear to match the result of your paper-and-pencil calculations. Rewrite the calculator result in terms of natural logarithms to verify that your paper-and-pencil calculations are correct.

Problem 3 – Investigating a Tangent to the Graph of $y = \ln x$

Advance to page 3.2.

This page shows the graph of $y = \ln x$ and the point (e, 0) located on the x-axis. Construct a line perpendicular to the x-axis through (e, 0). Use the **Intersection Point(s)** tool to place a point where the perpendicular line intersects $y = \ln x$. Finally, select the **Tangent** tool and click once on the intersection point to construct the tangent to $y = \ln x$ at $x = e$.

- What do you notice about this tangent line? *(Hint: You may find it helpful to extend the tangent line.)*

- What is the exact slope of this tangent line? *(Hint: You may find it helpful to select the tangent and select **Coordinates and Equations** to obtain its equation which will display an approximation of its slope.)*