The Derivative of Logs

ID: 9093

Time required
45 minutes

Activity Overview

Students will use the graph of the natural logarithm function to estimate the graph of the derivative of this function. They will do this by inspecting the slope of a tangent to the graph of the natural logarithm function at several points and using this information to construct a scatter plot for the derivative of the y = ln x. Students will then generalize their findings for any logarithmic function. Finally, students will investigate an interesting property related to a tangent to y = ln x.

Topic: Formal Differentiation

- *Derive the Logarithmic Rule and the Generalized Logarithmic Rule for differentiating logarithmic functions.*
- *Use **Limit** to show* $\lim_{h\to 0}\frac{ln(a+h)-ln(a)}{h}=\frac{1}{a}$ *and verify the Logarithmic Rule for differentiation.*
- *Graph the functions f(x) =* e^x *and g(x) = ln(x) and verify that they are inverse functions.*

Teacher Preparation and Notes

- *Students should know the definition of the derivative of a function.*
- *Students should be familiar with the change-of-base formula. Specifically, they should be able to rewrite a logarithmic expression in base e.*
- *As an extension to this activity, have students prove that the derivative of y = ln(x) is* $y=\frac{1}{x}$*. They can do this by rewriting y = ln(x) as* $e^y = x$ *and using implicit differentiation.*
- *This activity is designed to be student-centered with the teacher acting as a facilitator while students work cooperatively.*
- *Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.*
- ***To download the student TI-Nspire document (.tns file) and student worksheet, go to education.ti.com/exchange and enter "9093" in the keyword search box.***

Associated Materials

- *DerivativeOfLogs_Student.doc*
- *DerivativeOfLogs.tns*

Suggested Related Activities

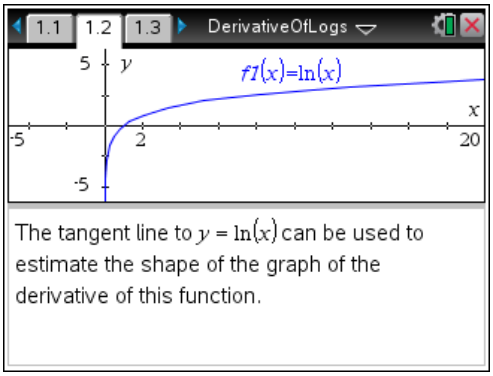
To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Logging In (TI-Nspire technology) — 12179*
- *Implicit Differentiation (TI-Nspire technology) — 8970*
- *Investigating the Derivatives of Some Common Functions (TI-84 Plus family) — 4368*

Introduction

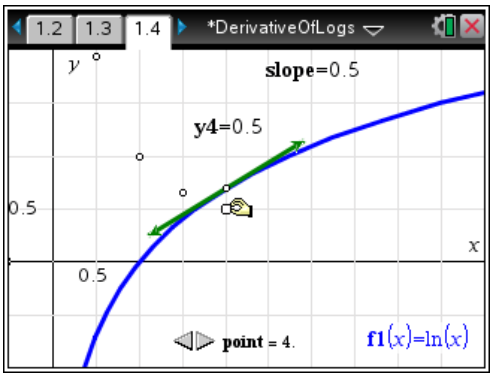
The tangent line to $y = \ln(x)$ can be used to estimate the shape of the graph of the derivative of this function.

Use page 1.2 to explain to students that they will be graphing a scatterplot that will represent the derivative of the $y = \ln(x)$. Students will use the slope of the tangent to the graph of $y = \ln(x)$ as the y-coordinate of each point on the scatterplot. The x-coordinate will be the same as the x-coordinate of the point of tangency.



Problem 1 – Finding the derivative of $y = \ln(x)$

Students will use the slider to select each of nine points along the x-axis. Students will then drag the tangent line along the graph so that it is directly above or below each of the points on the x-axis. Next, they will move the corresponding x-axis point to a location such that the y-coordinate is approximately equal to the slope of the tangent. In the diagram at right, it can be seen that the fourth point on the x-axis will need to be moved such that its y-coordinate, **y4**, is approximately equal to 0.5.

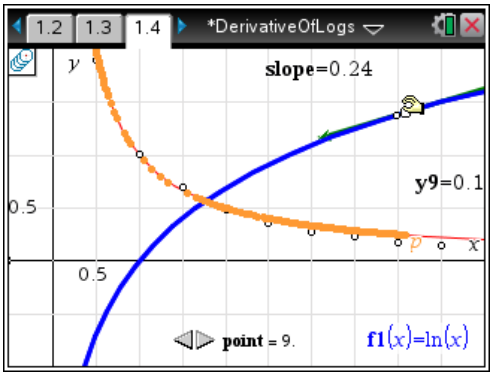


TI-Nspire Navigator Opportunity: *Class Capture*

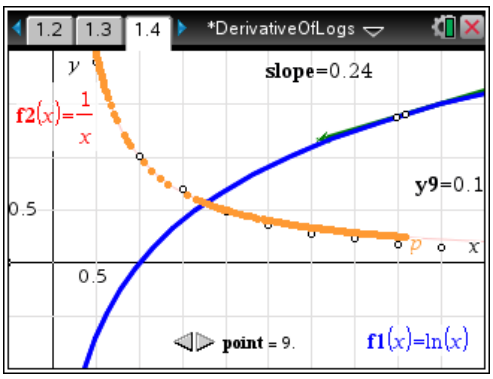
See Note 1 at the end of this lesson.

To check the accuracy of the scatter plot, students can select the **Hide/Show** tool from the Actions menu and click once on the point labeled p. If students cannot see this point, have them move the tangent line a bit—this will likely bring point p into view. (p appears in orange on TI-Nspire CX and CX CAS). Then students will select the tangent point and move it along the ln x curve,

Point p represents the derivative of $y = \ln x$ that corresponds to the current point of tangency. Students are to use **Geometry Trace** to see the graph of the derivative traced out.



Students are asked to guess the function represented by the scatter plot. If they are having trouble finding this function, mention that the derivative *decreases* as *x* increases and *increases* as *x* decreases. This should help them recognize that the derivative can be represented by the inverse relationship $y = \frac{1}{x}$.

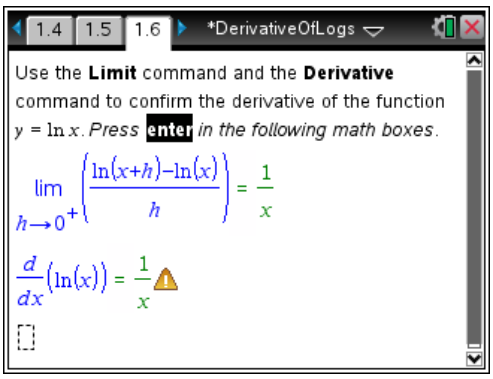


TI-Nspire Navigator Opportunity: *Live Presenter*

See Note 2 at the end of this lesson.

Students are to confirm their result by evaluating a limit expression on page 1.6.

The **derivative** command provides a final check of this result.



Problem 2 – Finding the derivative of any logarithmic function $y = \log_b x$

Students will use the change-of-base formula to find the derivative of $\log_b x$ as shown here.

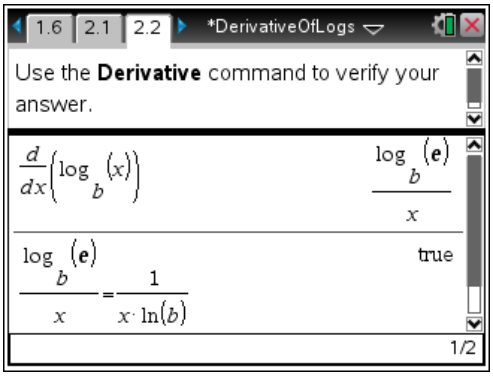
$$\log_b x = \frac{\ln x}{\ln b} \qquad \text{Change-of-base formula}$$

$$\frac{d}{dx}(\log_b x) = \frac{d}{dx}\left(\frac{\ln x}{\ln b}\right)$$

$$\frac{d}{dx}(\log_b x) = \frac{1}{\ln b} \cdot \frac{d}{dx}(\ln x) \qquad \frac{d}{dx}(c \cdot f(x)) = c \cdot \frac{d}{dx}(f(x))$$

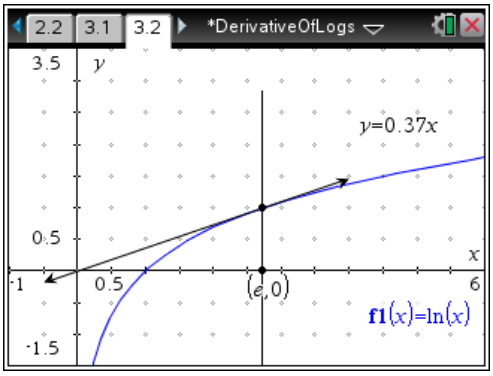
$$\frac{d}{dx}(\log_b x) = \frac{1}{x \cdot \ln b} \qquad \frac{d}{dx}(\ln x) = \frac{1}{x}$$

Students will check their paper-and-pencil calculations using the **Derivative** command. The first line shown on the screen at right shows that the calculator expresses the derivative of $\log_b x$ as $\frac{\log_b e}{x}$. The second line shows that this form of the derivative is equal to the form found using paper-and-pencil calculations.



Problem 3 – Investigating a Tangent to the Graph of *y* = ln *x*

Students are to construct the tangent to $y = \ln x$ at the point where x is equal to e. After extending the tangent line (as shown in the screen at right), students will notice that it passes through the origin. They will also see from the right triangle that the slope of the tangent is equal to $\frac{1}{e}$ or approximately 0.37.



TI-Nspire Navigator Opportunity: *Live Presenter*

See Note 3 at the end of this lesson.

TI-Nspire Navigator Opportunity: *Quick Poll*

See Note 4 at the end of this lesson.

TI-Nspire Navigator Opportunities

Note 1
Problem 1, Class *Capture*
This would be a good time to do a Class Capture to verify students are manipulating the slider correctly and positioning the moveable points correctly.

Note 2
Problem 1, *Live Presenter*
This would be a good opportunity to select a student to be a Live Presenter, particulary if other students are struggling finding the hidden point, *p*, or performing the Geometry Trace.

Note 3
Problem 3, *Live Presenter*
This would be a good opportunity to select another student to be a Live Presenter to help demonstrate to others how to construct the perpendicular line, label the intersection point, and construct the tangent line at that point.

Note 4
Problem 3, *Quick Poll*
You may choose to use Quick Poll to assess student understanding. You may ask the students to state the exact value of the slope of the contructed tangent line.

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