

Nspire Activity: It's Getting Crowded!

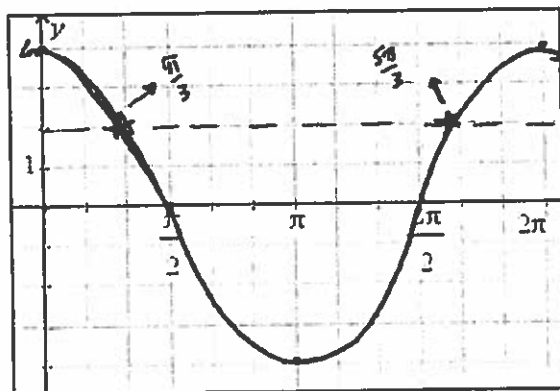
In this activity you will gain an understanding for how to solve more complex trig equations by investigating solutions to equations graphically.



Part 1: Starting Simple

$$\rightarrow \cos x = \frac{1}{2}$$

- Solve the trig equation $2 = 4\cos(x)$. List all solutions over the interval $[0, 2\pi)$. $\{\frac{\pi}{3}, \frac{5\pi}{3}\}$
- Show the solutions to $2 = 4\cos(x)$ graphically by plotting $y = 2$ and $y = 4\cos(x)$ on the grid below. Mark all solutions/intersections with a ★.



- Open the document titled 'solutions' and move to page 1.2. You will see the graph of $y = 2$ and $y = 4\cos(ax)$, where 'a' is a value attached to a slider (currently set to 'a' = 1). Use the graph on page 1.2 to check your solutions from above. Then, list your solutions from step #1 (in numerical order) in the first column of the tables in steps #6 and 8 on the next page.

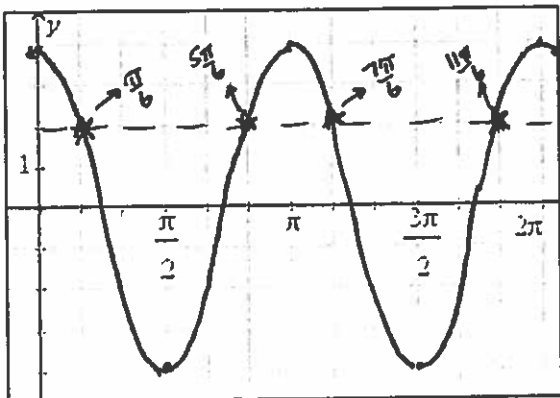
Part 2: Making Connections to Solutions of the equation $2 = 4\cos(2x)$

- In the next step, you will determine solutions to the equation $2 = 4\cos(2x)$ by observing the intersections of the graphs of $y = 2$ and $y = 4\cos(2x)$. First, answer the questions below.

What is the period of the graph of $y = 4\cos(2x)$? π

How many cycles of the graph of $y = 4\cos(2x)$ will be seen over the interval $[0, 2\pi)$? 2

- Change the slider so that the value of 'a' is equal to 2 and list all solutions to the equation displayed over the interval $[0, 2\pi)$ from smallest to largest. Then, make a sketch of the graph, marking all solutions with a ★.



Equation: $2 = 4\cos(2x)$

Solutions: $\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}\}$

6. List the 1st and 2nd solutions from step #5 (in order) in the 2nd column. Then, list the 3rd and 4th solutions (in order) from step #5 in the 3rd column.

1 st column	2 nd column	3 rd column
Solutions to $2 = 4\cos(x)$	Solutions to $2 = 4\cos(2x)$	
$\frac{\pi}{3}$	$\frac{\pi}{6}$	$\frac{7\pi}{6}$
$\frac{5\pi}{3}$	$\frac{5\pi}{6}$	$\frac{11\pi}{6}$

Compare the corresponding solutions in the 1st and 2nd columns. Explain what you observe.

Solutions to $2 = 4\cos(2x)$ are $\frac{1}{2}$ of solutions to $2 = 4\cos x$.

2nd column = 1st column

Compare the corresponding solutions in the 2nd and 3rd columns. Explain what you observe.

Solutions in the 3rd column are π more than solutions in 2nd column.

What do you think accounts for the observations you made when comparing the solutions?

The period of $y = 4\cos(2x)$ is half of $y = 4\cos(x)$, so solutions occur in ' $\frac{1}{2}$ of the time' and additional solutions occur 1 period (π) after the initial solutions.

Part 3: Predicting solutions to the equation $2 = 4\cos(3x)$

7. In the next step, you will predict solutions to the equation $2 = 4\cos(3x)$ and then check them by looking at the intersections of the graphs of $y = 2$ and $y = 4\cos(3x)$. First, answer the questions below.

What is the period of the graph of $y = 4\cos(3x)$? $\frac{2\pi}{3}$

How many cycles of the graph of $y = 4\cos(3x)$ will be seen over the interval $[0, 2\pi)$? 3

8. Using what you have learned in Part 2, predict solutions to the equation of $2 = 4\cos(3x)$. List all solutions and explain how you determined the solutions.

Solutions to $2 = 4\cos(x)$	Solutions to $2 = 4\cos(3x)$		
$\frac{\pi}{3}$	$\frac{\pi}{9}$	$\frac{7\pi}{9}$	$\frac{13\pi}{9}$
$\frac{5\pi}{3}$	$\frac{5\pi}{9}$	$\frac{11\pi}{9}$	$\frac{17\pi}{9}$

Explanation: The two initial solutions ($\frac{\pi}{9}, \frac{5\pi}{9}$) occur in ' $\frac{1}{3}$ of the time' of the solutions to $2 = 4\cos x$ and additional solutions occur 1 period ($\frac{2\pi}{3}$) after initial solutions

9. Now, change the 'a' slider so that is equal to 3 and observe the solutions to $2 = 4\cos(3x)$ graphically. Compare the actual results to your prediction above.