# Give Me Five!

## TI-NSPIRE™ CX II

TEACHER NOTES

IB Aligned

## Math Objectives

- Students will organize data and find the Five Number Summary.
- Students will use their handhelds to verify the data analysis that have produced by hand.
- Students will interpret their data analysis using the visual of a Box and Whisker diagram.
- Students will try to make a connection with how to understand these topics in IB Mathematics courses and on their final assessments.

## Vocabulary

- Outlier
- Quartile
- Inter-Quartile Range
- Five Number Summary
- Box and Whisker Diagram

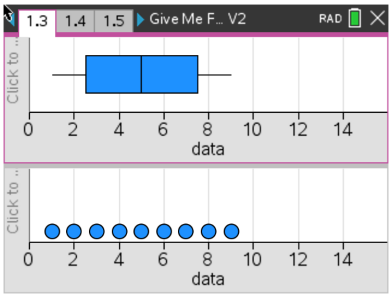
## About the Lesson

- This lesson is aligning with the curriculum of IB Mathematics Applications and Interpretations SL/HL and IB Mathematics Approaches and Analysis SL/HL
- This falls under the IB Mathematics Content Topic 4 Statistics and Probability:
  - **4.1:** **(a)** Interpretation of Outliers
  - **4.2:** **(a)** Presentation of Data (discrete and continuous)
    **(d)** Production and understanding of box and whisker diagrams
  - **4.3**: **(a)** Measure of central tendency (mean, median, and mode)
    **(c)** Measures of dispersion (Range and Inter-Quartile range)

  As a result, students will:
  - Apply this information to real world situations.

## TI-Nspire™ Navigator™

- Transfer a File.
- Use Class Capture to examine patterns that emerge.
- Use Live Presenter to demonstrate.
- Use Teacher Edition computer software to review student documents.
- Use Quick Poll to assess students' understanding




**Tech Tips:**

- This activity includes screen captures taken from the TI-Nspire CX II handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

**Lesson Files:**

*Student Activity*

Nspire-GiveMeFive-Student.pdf
Nspire-GiveMeFive-Student.doc
Give Me Five.tns

## Activity Materials

Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software

In this activity, students will discuss and describe the center and spread of a univariate data set by way of a five number summary and visually by a box & whisker diagram. Students will then apply this knowledge to real life applications to enhance their ability to understand this math in statistical data analysis.

**Teacher Tip:** The file to download to the handheld is a preset tool that will help guide your students through the Five Number Summary and gives nice visuals throughout the pages, but feel free to lead your students to enter the data on their own without the help of the tools provided in the preset document.

### Introduction

A univariate set of data is a list of numbers that describes the different value of a variable characteristic across a range of different units. For example, if a study involved finding out the height of a range of people, each person whose height is measured is statistically considered to be a 'unit'. Height is the characteristic that varies (variable) and the list of height measurements is called the data.

When describing a group of data, there are generally two main types of things to consider:

a) Measure of center – this is a single value that could be used as a representative of the entire data set (e.g. mean, median, mode)

b) Measure of spread – this is a number that indicates how spread out the data are (e.g. standard deviation, range, inter-quartile range)

### Problem 1 – The Fantastic Five

A five number summary is a convenient way of describing a set of data as it provides us with information about both center and spread. Consider the set of data: {1, 2, 3, 4, 5, 6, 7, 8, 9}. We can see that the numbers are already ordered from lowest to highest.

1. Find the *minimum* value in the data set. We call this value **MinX**.

   **Solution: 1**

2. Find the *maximum* value in the data set. We call this value **MaxX**.

   **Solution: 9**

3. Find the *middle* value in the data set. We call this value **MedianX**.

**Solution: 5**

4. Look at the numbers that are less than the Median, find the median of *this* set of numbers. Discuss with a classmate what the median would be if this data set was an even number of data and if this data set was an odd number of data. Find the name of this piece of data.

**Solution: 2.5**

As there are only four numbers in this group, you have to work out what number is exactly half way between two digits, for this data set it will be a fraction, we call this value **$Q_1$** or the **Lower Quartile**.

5. Look at the numbers that are less than the Median, find the median of *this* set of numbers. Discuss with a classmate what the median would be if this data set was an even number of data and if this data set was an odd number of data. Find the name of this piece of data.

**Solution: 7.5**

As there are only four numbers in this group, you have to work out what number is exactly half way between two digits, for this data set it will be a fraction, we call this value **$Q_3$** or the **Upper Quartile**.

6. You have found $Q_1$ and $Q_3$. Discuss with a classmate what $Q_2$ is.

**Solution:** $Q_2$ is the median.

The five numbers that are your answers to questions 1 to 5 are called the **five number summary**. Usually the five number summary is written in the order MinX, $Q_1$, Median, $Q_3$, MaxX. The median (Answer to question 3) is the measure of center. The other numbers provide indications of spread.

- MaxX minus MinX is the **Range**.
- $Q_3$ minus $Q_1$ is the **Inter-Quartile Range (IQR)**.

7. Find the *range* for the data set {1, 2, 3, 4, 5, 6, 7, 8, 9}.

**Solution: 8**

8. Find the *Inter-Quartile Range* for the data set {1, 2, 3, 4, 5, 6, 7, 8, 9}.

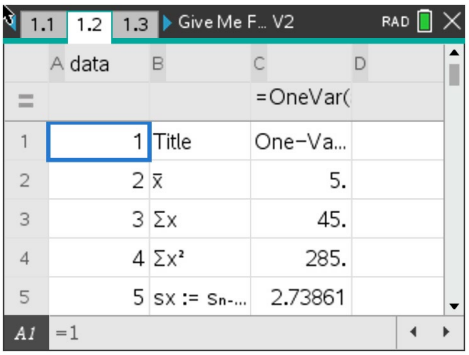
**Solution: 5**

**Problem 2 – Automatic Calculation of the Five Number Summary**

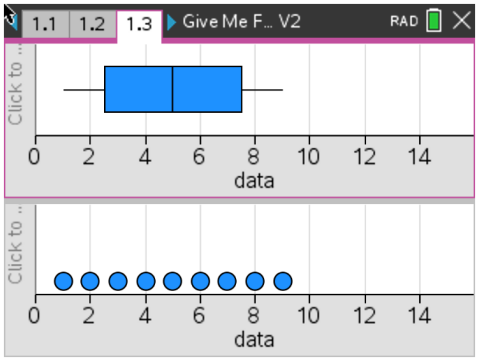
Open the *tns* file: Give Me Five.tns

Observe the data set {1, 2, 3, 4, 5, 6, 7, 8, 9} on page 1.2.

The data values are entered into a Lists & Spreadsheets page under the column title of 'data'. Summary statistics for the data have been calculated and displayed in the next two columns.




Page 1.3 provides a graphical representation of the data set. This type of graph is called a Box Plot or a Box & Whisker Diagram. A dot plot is included beneath the Box Plot.
If you scroll your calculator cursor over this graph, the five numbers of the five number summary will be revealed. Note that they are in line with corresponding numbers on the scale below.




Move the cursor over the top of the data point with the largest value in the dot plot (9). Grab the point (click and hold) and move it to the right.

**Tech Tip:** You can navigate between the two Data and Statistics applications by using **Ctrl + Tab**, similar to ALT + Tab on a Windows™ platform.

1. Explain why $Q_1$, the median and $Q_3$ do not change when the data point (9) is increased.

**Solution:** The 9 is in a different quartile and remains in the same quartile even when it is increased.

2. Explain what happens to the whisker when this data point is moved further and further away from the rest of the data. Find at what value, approximately, this significant change occurs.

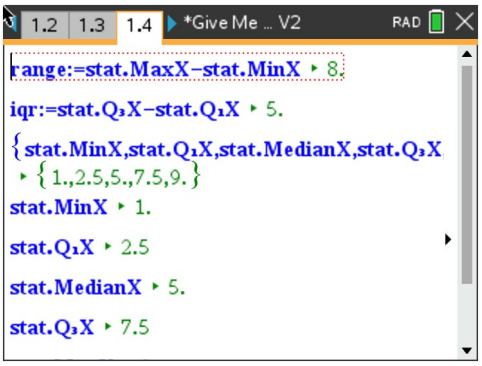
**Solution:** The length of the whisker increases until it 'snaps' and the point becomes an outlier … just past 15.

Navigate to page 1.4. This is a Dynamic Notes page that is set up to show the **range** and the **interquartile range** of the data set from page 1.2.

It shows the five number summary as a set of 5 values in brackets { }.

The five values are also shown on separate lines below this.

**Note that if you go back to page 1.2 and make changes to the data set, the Box Plot and the Dynamic Notes Five Number Summary will automatically change to reflect the new data set.**




3. Navigate to page 1.2 and replace the data set {1, 2, 3, 4, 5, 6, 7, 8, 9} with {9, 3, 8, 5, 7, 4, 1, 6, 2}. Discuss with and state the affects this may have on the Box Plot and statistics calculations. Explain why you think this is so.

**Solution:** It doesn't make any difference as the values haven't changed, they're just re-ordered. The five number summary is based on the ordered values therefore they will be returned to their corresponding order before calculation.

4. Use the automatic calculator (Page 1.4) to compute the five number summary of the data set: {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}. Validate the five number summary by hand.

**Solution:** MinX = 1, $Q_1 = 3$, MedX ($Q_2$) = 5.5, $Q_3 = 8$, MaxX ($Q_4$) = 10
(by inspection) {1, 2, **3**, 4, 5} $\frac{5+6}{2} = 5.5$ {6, 7, **8**, 9, 10} (by inspection)

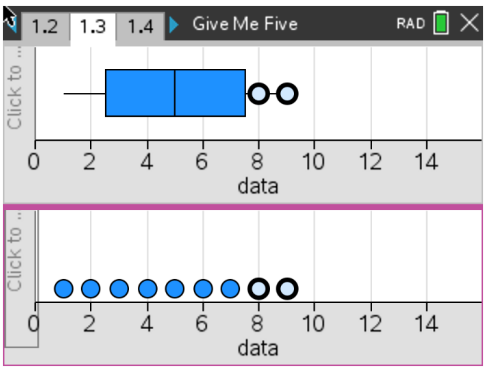
5. Use your answer to the previous question to find a data set that has a five number summary made up entirely of integers.

**Solution:** Answers may vary.
Examples: {1, 2, 3, 4, 5, 5, 7, 8, 9} or {1, 2, 3, 4, 6, 6, 7, 8, 9}

**Problem 3 – Consideration of shape and skew**

So far the data sets we have considered have been **symmetrical**. That is, the Box Plot is geometrically symmetrical and has a vertical line of symmetry at the median. This means that $Q_1$ is as far below the median as $Q_3$ is above it and MinX is as far below the median as MaxX is above it. You may also have noticed that the **mean** value (indicated in column C of 1.2) is always the same as the median.

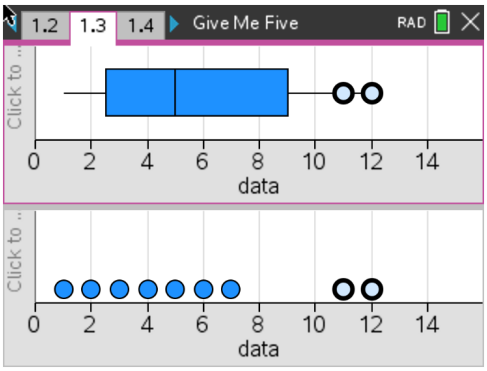
If necessary, adjust the data set on Page 1.2 so that it is equal to: {1, 2, 3, 4, 5, 6, 7, 8, 9}. Note that the median value is 5, as also is the mean. Now move to Page 1.3 and observe the Box Plot for these data, it is perfectly symmetrical. Click on the MaxX value at the end of the right-hand whisker. The MaxX value (9) will be highlighted and also the value (8).



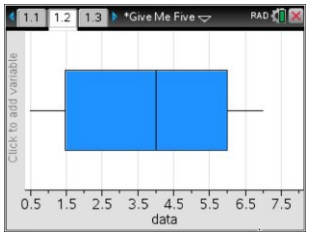
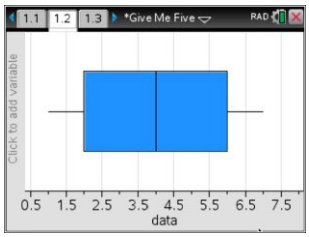
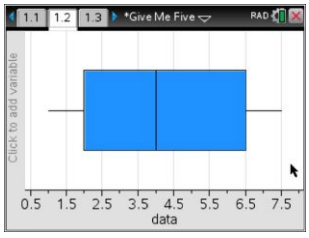

Drag the whisker to the right so that MaxX is at about 12.

$Q_3$ will move up to about 9. Notice that the distribution is now no longer symmetrical. The part of the box that is between the median and $Q_3$ is bigger than the part between the median and $Q_1$. The distribution is now said to be **positively skewed** or **skewed right**.

Notice also that, although the median is still 5, the mean value has moved up to about 5.6. Restore the original data set and repeat this process to show a **negative skew** or **skew left**.



1. Match each of the following Box Plots with its matching description of symmetry and comment about measure of center.

| Box Plot | Description of Shape | Comment on measure of center |
|---|---|---|
|  | Symmetrical | Mean > Median |
|  | Positively skewed | Mean < Median |
|  | Negatively skewed | Mean = Median |

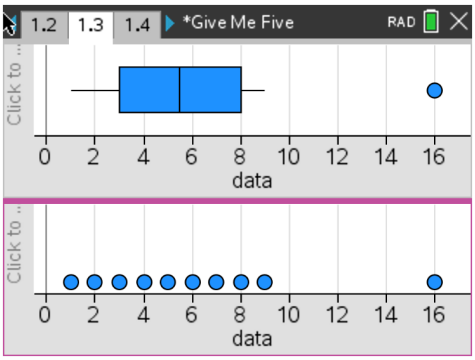


**Teacher Tip:** This would be a good point to go a little further with this problem. You can have wonderful discussion about each type of skew and ask the students to give real life examples for each.

**Outliers and Fences**

Restore your data list to the set: {1, 2, 3, 4, 5, 6, 7, 8, 9}.

Now add in a 10th value to the list. Make this value 16. Observe the corresponding Box Plot. Notice that the value 16 doesn't appear within the main box or whisker, but is shown as a dot on its own. This is because the score 16 is so far away from the other data points that it is considered to be an **outlier**.




Experiment by replacing the 16 with values that are closer to the original data set. Try replacing it with 14, 13, 12, 11, 10.

A numerical value that determines the threshold for outliers can be computed and is referred to as the **upper fence value** where the outlier is above the median and **lower fence value** where the outlier is below the median. The upper and lower fences are defined by using the Inter-Quartile Range (IQR).

$$\text{Upper Fence Value} = Q3 + 1.5 \times IQR$$

$$\text{Lower Fence Value} = Q1 - 1.5 \times IQR$$

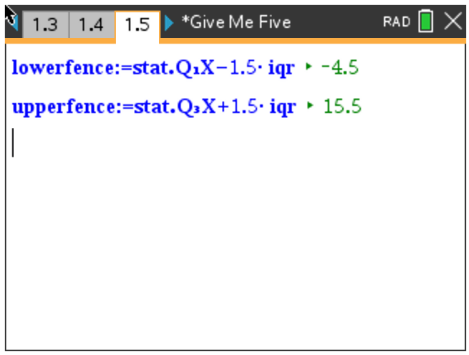
**Example**

If you have a set of 8 scores {1, 2, 3, 4, 5, 6, 7, 8}, such that $Q_1 = 2.5$, $Q_3 = 6.5$ and the IQR = 4.

Upper fence = 6.5 + 1.5 x 4
= 12.5

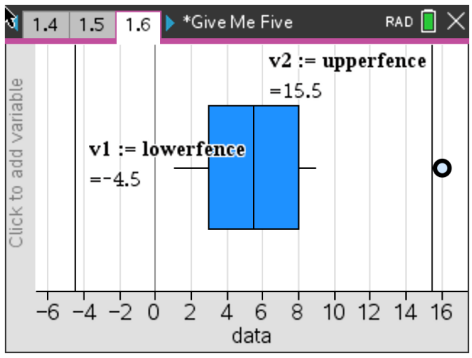
Lower Fence = 2.5 – 1.5 x 4
= -3.5

Page 1.5 is a Dynamic Notes page that shows the calculation of Lower and Upper Fences.




Page 1.6 Shows the Box Plot again, this time with the fences displayed as vertical lines.

Drag the outlier to the fence and observe what happens.




2. Discuss with a classmate and explain how it is possible to calculate the IQR whilst a single outlier is changed.

**Solution:** As $Q_1$ and $Q_3$ remain unchanged when the outlier is changed, the IQR will also remain unchanged.

**Further IB Application**

The scores of a mathematics test given to period 1 are shown below.

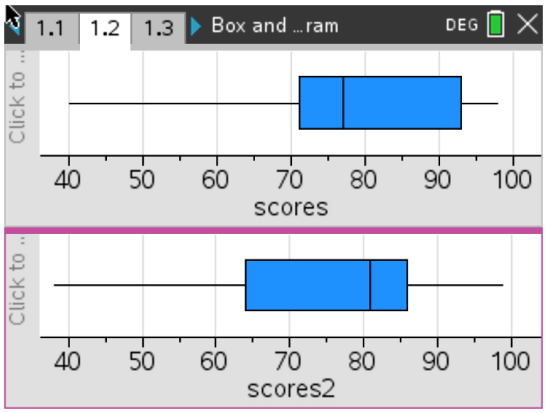
40, 62, 65, 71, 73, 74, 75, 77, 80, 90, 92, 93, 96, 97, 98

For the data, the lower quartile is 71 and the upper quartile is 93.

(a) Show that the test score of 40 would not be considered an outlier.

**Solution:** (93 – 71) x 1.5 or 22 x 1.5 seen anywhere or 33 seen anywhere
71 – 33
38
40 > 23
So is not an outlier

The same mathematics test was given to period 2 and the box and whisker diagram showing their scores (**scores2**) and comparing them to the scores of period 1 (**scores**) are below.




A fellow mathematics teacher looks at the box and whisker diagrams and believes that period 2 performed better than period 1.

(b) Using the diagrams above, state one reason that may support the mathematics teacher's opinion and one reason that may counter it.

**Solution:** The median score for the second period class is higher than the median score for the first period class.
Then:
But the scores are more spread out in the second period class.
Or
But the scores are more inconsistent in the second period class.
Or
But the lowest scores are in the second period class.
Or
But the lower quartile is lower in the second period class.

**Teacher Tip:** This is a good place to have students discuss this situation and see if they can add more questions, scenarios and discussions to the problem.

**TI-Nspire Navigator Opportunity: *Quick Poll (Open Response)***
**Any part to any Problem in the activity would be a great way to quickly assess your student's understanding of finding and discussing the Five Number Summary and Box and Whisker Plots.**

**Teacher Tip:** Please know that in this activity there is a lot of time dedicated to students talking with one another and sharing their thoughts with the class. The goal here is to not only review the Five Number Summary, but also to generate discussion.

***Note: This activity has been developed independently by Texas Instruments and aligned with the IB Mathematics curriculum, but is not endorsed by IB™. IB is a registered trademark owned by the International Baccalaureate Organization.*