EXPLORATIONS

Chapter 13

Features Used

crossP( ), dotP( ), real( ), conj( ), [i], NewProb, [STO►], Polar graphs

Setup

[◆] 1
NewFold ant
setMode("Complex Format", "Polar")

Antennas

This chapter describes how to perform basic antenna and radiation calculations with the TI-89. Antenna patterns, radiation resistance, radiation integrals, and phased array patterns are included.

Topic 62: Incremental Dipole

The most fundamental antenna is the incremental dipole as pictured in Figure 1.

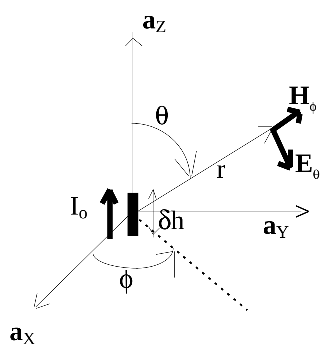


Figure 1. Incremental dipole

For mathematical convenience, the dipole is centered at the origin and aligned with the z-axis. It has a length δh which is much shorter than the wavelength λ, that is, δh<<λ. It is excited by a sinusoidal current source of angular frequency ω. The resulting current is uniformly distributed along the dipole and has a phasor form of i_σ.

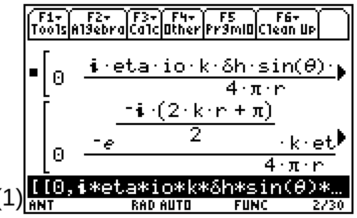
Example 1: Calculating Incremental Dipole Values

The far-zone electric field radiated by the incremental dipole is given by the vector

$$\mathbf{E} = j\eta \frac{io k \delta h \sin\theta}{4\pi} \frac{e^{-jkr}}{r} \mathbf{a}_\theta \, V/m$$

1. Clear the TI-89 by pressing [2nd] [F6] **2:NewProb** [ENTER].

   Using [MODE], set **Complex Format** to **POLAR**.



(1)

Note: To enter δ, press [◆] [(] [alpha] **d** *on the keyboard.*

2. Enter the vector in spherical coordinates as shown in screen 1.

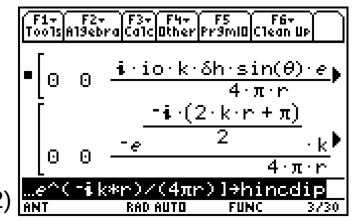
   [2nd] [[] **0** [,] [2nd] [i] **eta** [×] **io** [×] **k** [×] [◆] [(] [alpha] **d h** [×] [2nd] [SIN] [◆] [θ] [)] [◆] [e^x] [(-)] [2nd] [i] **k** [×] **r** [)] [÷] [(] **4** [2nd] [π] **r** [)] [,] **0** [2nd] []] [STO▸] **eincdip**

3. The far-zone magnetic field is

$$\mathbf{H} = j\frac{io k \delta h \sin\theta}{4\pi}\frac{e^{-jkr}}{r}\mathbf{a}_{\phi} \mathrm{A/m}$$

   Enter the vector as shown in screen 2.

   [2nd] [[] **0** [,] **0** [,] [2nd] [i] **i o** [×] **k** [×] [◆] [(] [alpha] **d h** [×] [2nd] [SIN] [◆] [θ] [)] [◆] [e^x] [(-)] [2nd] [i] **k** [×] **r** [)] [÷] [(] **4** [2nd] [π] **r** [)] [2nd] []] [STO▸] **hincdip**



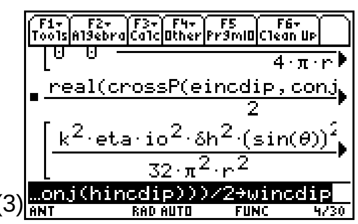
(2)

4. These fields represent outward propagating, spherical waves with an amplitude that varies with polar angle. The fields decrease as the distance to the antenna is increased. The power density of such a field is given by

$$\mathbf{W} = \frac{\mathrm{Re}(\mathbf{E} \times \mathbf{H}^{*})}{2} \ \mathrm{w/m^2}$$

   Calculate the power density as shown in screen 3.

   [CATALOG] **real(** [CATALOG] **crossP(eincdip** [,] [CATALOG] **conj(hincdip** [)] [)] [)] [÷] **2** [STO▸] **wincdip**



(3)

   The result is

$$\left[\frac{k^2 eta(io)^2 \delta h^2 (\sin(\theta))^2}{32\pi^2 r^2} \quad 0 \quad 0\right]$$

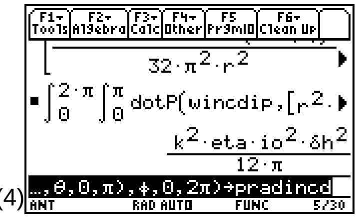
   This shows that the power is directed radially outward.

5. The total power radiated is calculated as

$$P_{RAD} = \oiint_{SPHERE} \mathbf{W} \bullet \mathbf{ds}$$

Enter this expression as shown in screen 4.

[2nd] [∫] [2nd] [∫] [CATALOG] **dotP(wincdip** [,] [2nd] [[] **r** [^] **2** [×] [2nd] [SIN] [◆] [θ] [)] [,] **0** [,] **0** [2nd] []] [)] [,] [◆] [θ] [,] **0** [,] [2nd] [π] [)] [,] [◆] [(] [alpha] **f** [,] **0** [,] **2** [2nd] [π] [)] [STO▸] **pradincd**



(4)

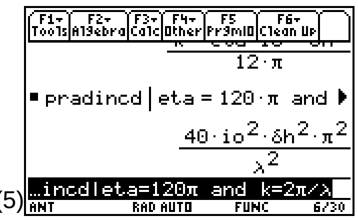
Note: To enter ϕ, press [◆] [(] [alpha] **f**.

Calculation of the radiated power is simplified since there are only radial components of both **W** and **ds**. Note that the differential surface element on the sphere is $r^2 \sin\theta d\theta d\phi$.

6. Substituting of the free space wave impedance as **eta**=120π and the wave propagation constant as **k**=2π/λ gives the average power radiated into free space.

Use the "with" operator to substitute the values as shown in screen 5.

pradincd [|] **eta** [=] **120** [2nd] [π] [CATALOG] **and k** [=] **2** [2nd] [π] [÷] [◆] [(] [alpha] **l**



(5)

Note: To enter λ, press [◆] [(] [alpha] **l**.

7. The current flowing into one side and out of the other side of the dipole is **io**. From a circuit point of view, the power extracted from the current by radiation is given by

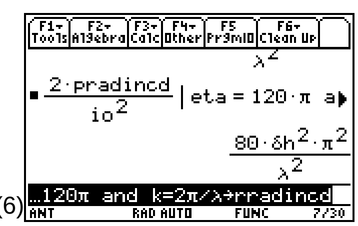
$$P_{RAD} = \frac{|io|^2 R_{RAD}}{2}$$

where R_{RAD} represents equivalent resistance to dissipate this power. The radiation resistance is calculated as

$$R_{RAD} = \frac{2P_{RAD}}{|io|^2}$$

Find the radiation resistance as shown in screen 6.

2pradincd [÷] **io** [^] **2** [|] **eta** [=] **120** [2nd] [π] [CATALOG] **and k** [=] **2** [2nd] [π] [÷] [◆] [(] [alpha] **l** [STO▸] **rradincd**

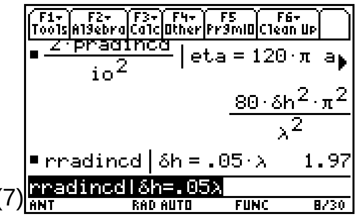


(6)

8. Calculate the radiation resistance for **δh**=0.05λ as shown in screen 7.

rradincd [|] [◆] [(] [alpha] **d h** [=] **.05** [◆] [(] [alpha] **l**

The result of R_{RAD} = 1.97 Ω is quite small for an incremental dipole.

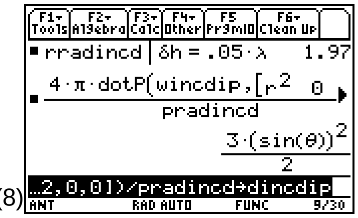
(7) 

9. The directive gain of an antenna is defined as

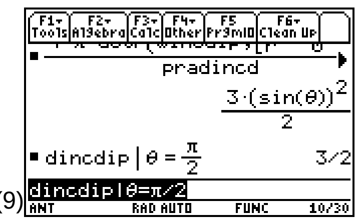
$$D_G = \frac{r^2W_r}{r^2W_{rAVE}} = 4\pi\frac{r^2W_r}{P_{RAD}}$$

Calculate the directive gain as shown in screen 8.

4 [2nd] [π] [CATALOG] **dotP(wincdip** [,] [2nd] [[] **r** [^] **2** [,] **0** [,] **0** [2nd] []] [)] [÷] **pradincd** [STO▸] **dincdip**

(8) 

10. The directive gain indicates how much the power density is increased or focused at an angular position compared to that of an isotropic source with the same radiated power. Directivity is the peak directive gain, that is, 3/2 for the infinitesimal dipole as shown in screen 9.

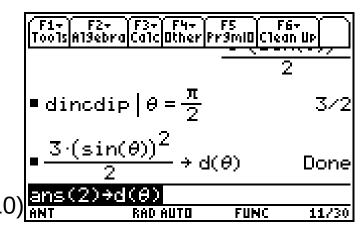
(9) 

Topic 63: Antenna Patterns

The directive gain or just gain shows the variation of power density with the polar angle θ at a fixed radial distance from the dipole. A plot of this variation, called the antenna power pattern and commonly normalized to one at the peak, helps to visualize this important characteristic of the dipole. It is standard practice to make pattern plots in the plane of the electric field, the E-plane pattern, and in the plane of the magnetic field, the H-plane pattern. The planes are chosen to include the peak value as well. For the dipole antenna, the peak occurs at θ=90° so the E-plane pattern is plotted as a function of θ in a plane of constant ϕ; the H-plane pattern is plotted as a function of ϕ in the θ=90° plane.

1. To use the polar graphing capabilities for an E-plane pattern, directive gain must be defined as a function of θ as shown in screen 10.

[2nd] [ANS] [←] [←] **2** [)] [STO▸] **d** [(] [◆] [θ] [)]

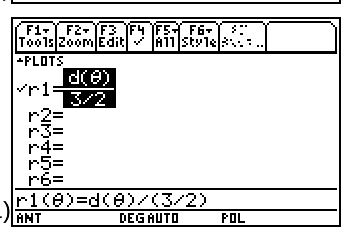
(10) 

2. Set the **Graph** mode to **POLAR** and the **Angle** mode to **DEGREE** using [MODE].

3. In the Y= Editor, define the function to be graphed (screen 11).

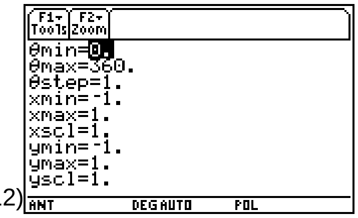
[◆] [Y=] [ENTER] **d** [(] [◆] [θ] [)] [÷] [(] **3** [÷] **2** [)]

Note that the division by 3/2 is to normalize the peak value of the graph to 1.

(11) 

4. In the Window Editor, set the window variable values (screen 12) for θ to vary from 0 to 360° (that is **θmin**=0 and **θmax**=360). Although polar angle θ is defined over 0 to 180°, this range is needed to include both half-planes on which ϕ is constant.

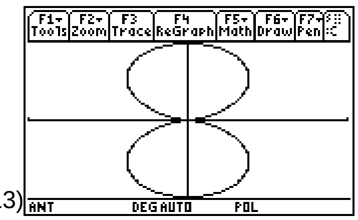
θstep=1 is used to generate the plot here, but **θstep**=5 is faster and good enough in most cases. Since the pattern is normalized to one (division by the 3/2 factor), the ranges on **x** and **y** are ±1.



(12)

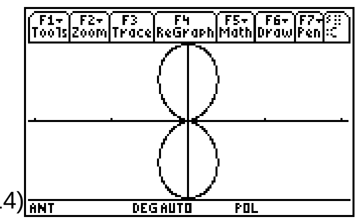
5. Press [◆] [GRAPH] to plot the E-plane pattern (screen 13).

This is a distorted view of the pattern due to different scaling on **x** and **y**.


(13)

6. Use [F2] **5:ZoomSqr** to plot a properly scaled pattern (screen 14).

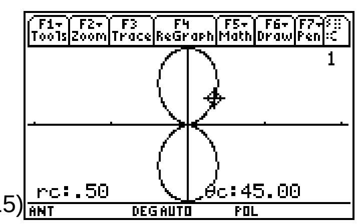
Since the polar angle θ is measured from the positive z-axis on the dipole and the graphing angle θ is measured from the positive x-axis on the screen, the dipole lies along the x-axis of the pattern.


(14)

7. Half-power beamwidth is a parameter used to describe antennas. It is the angular separation of the half-power points of a pattern. Use the trace cursor ([F3] and [◄], [►], [2nd] [◄], or [2nd] [►]) to display pattern values and angles and find the beamwidth.

First, use [◆] [I] [►] **2:Polar** to set coordinates in the polar mode.

The right-hand half-power point is at θ=45°. Press [F3] and **45** [ENTER]. Screen 15 shows that at an angle of 45° the power is 0.5.

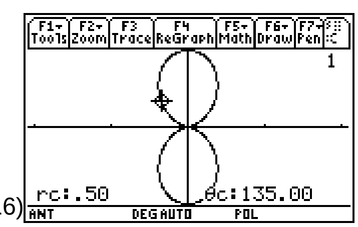


(15)

8. The left-hand half-power point is at θ=135°.

Press **135** [ENTER].

So the E-plane beamwidth is 135-45=90°.

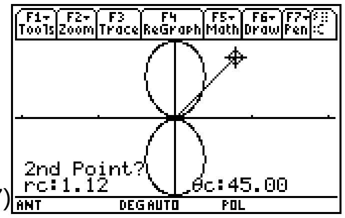


(16)

9. The concept of beamwidth can be further emphasized by using [2nd] [F7] **3:Line**.

10. In response to the on-screen question "**1st Point?**", position the cursor at the origin and press [ENTER].

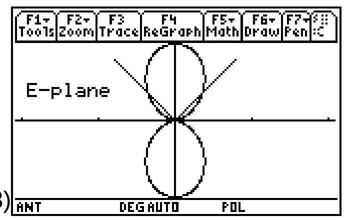
11. For "**2nd Point?**", position the cursor at the angle of the lower half-power point, θ=45°, with **rc**=1.12 and press [ENTER] (screen 17).

(17) 

12. Add a second line at the upper half-power point in the same manner.

13. Press [ESC] to exit this mode.

14. Add text to the graph using [2nd] [F7] **7:Text**. Position the cursor at the starting point (screen 18). If there is an error, use the eraser ([2nd] [F7] **2:Eraser** and [ENTER], then hold down the [↑] key to erase).

(18) 

15. The H-plane pattern is plotted as a function of ϕ; however, it has no ϕ dependence so it is a constant and graphs as a circle of radius 1.

Topic 64: Phased Arrays

Phased arrays are commonly used to tailor antenna patterns to a desired shape. When several identical elements are located near each other, they form an array. The pattern of the array is the product of an element factor, a geometric factor, and an array factor. The array factor, AF, for N identical, equi-amplitude radiators located on the z-axis with uniform spacing D, is given by

$$\mathrm{AF} = \left| \frac{\sin\left(\frac{N}{2}\left(2\pi\frac{D}{\lambda}\cos\theta + \beta\right)\right)}{N\sin\left(\pi\frac{D}{\lambda}\cos\theta + \frac{\beta}{2}\right)} \right|$$

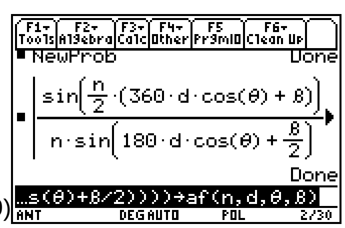
β is the progressive phase shift along the array from one element to the next. An alternate form allows graphing with θ in degrees and expresses radiator spacing, d=D/λ, in fractions of the wavelength for easy graphing of the array factor

$$\mathrm{AF} = \left| \frac{\sin\left(\frac{N}{2}(360d\cos\theta + \beta)\right)}{N\sin\left(180d\cos\theta + \frac{\beta}{2}\right)} \right|$$

1. Clear the TI-89 by pressing [2nd] [F6] **2:NewProb** [ENTER].

2. Enter the array factor expression as shown is screen 19.

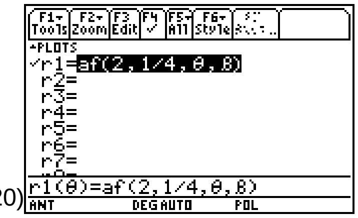
   [CATALOG] **abs(** [2nd] [SIN] **n** [÷] **2** [(] **360d** [×] [2nd] [COS] [◆] [θ] [)] [+] [◆] [(] [alpha] **b** [)] [)] [÷] [(] **n** [×] [2nd] [SIN] **180d** [×] [2nd] [COS] [◆] [θ] [)] [+] [◆] [(] [alpha] **b** [÷] **2** [)] [)] [)] [STO▸] **af** [(] **n** [,] **d** [,] [◆] [θ] [,] [◆] [(] [alpha] **b** [)]

(19) 

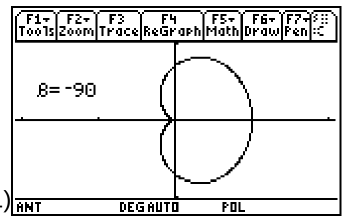
3. The beam from an array can be “steered” by varying the phase-shift between elements.

To graph the patterns for a two-element array (**n**=2) with **d**=1/4 spacing for phase shifts of β=-90°, 0°, 45°, and 90°, define -1 in the Y= Editor as shown in screen 20.

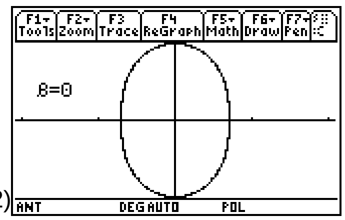
af (**2** , **1** ÷ **4** , ◆ [θ] , ◆ (alpha **b**)

(20)


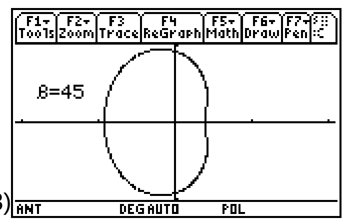
4. On the Home screen, assign the values -90, 0, 45, and 90 to **β.** Then graph each pattern as shown in screens 21 through 24.

(21)


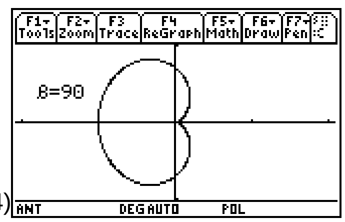
-90→β

(22)


0→β

(23)


45→β

(24)


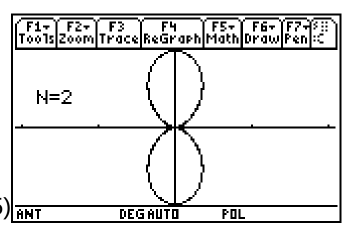
90→β

5. With more elements in an array, the beamwidth becomes narrower and more focused, as shown in screens 25-27 by arrays with 2, 4, and 6 elements (**n**=2, 4, and 6) spaced with one-half wavelength (**d**=1/2) and with zero phase-shift (β=0). However, this improvement in beamwidth is accompanied by an undesirable increase in the number and amplitude of sidelobes.

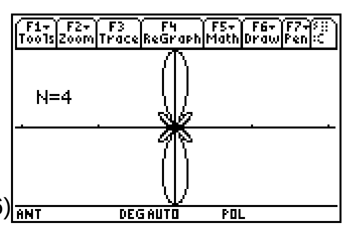
Edit **r1** in the Y= Editor.

af (**n** , 1 ÷ 2 , ◆ [θ] , **0**)

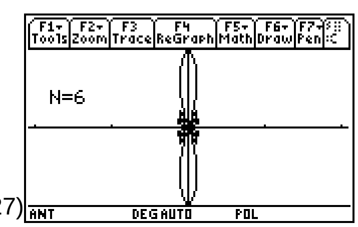
On the Home screen, assign the values of 2, 4, and 6 to **n**. Then graph each pattern.

(25)


2→**n**

(26)


4→**n**

(27)


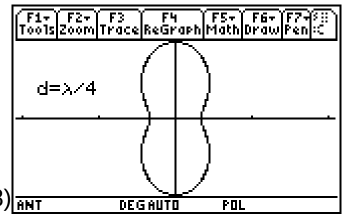
6→**n**

6. Finally, the element spacing can narrow the beamwidth, but the wider spacing causes more and larger sidelobes. This effect is shown in screens 28-30 by 3 elements (**n**=3) with spacing of λ/4, λ/2, and λ (**d**=1/4, 1/2, and 1) and **β**=0.

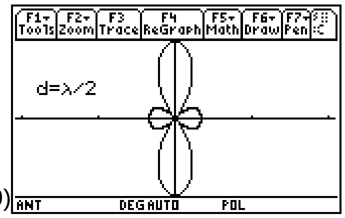
Edit **r1** in the Y= Editor.

af (**3** , **d** , ◆ [θ] , **0**)

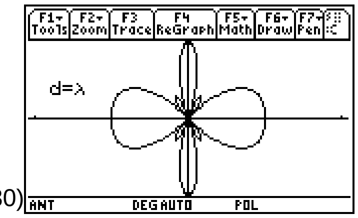
Assign the values for **d** on the Home screen. Graph the patterns.

(28) 

1/4→**d**

(29) 

½→**d**

(30) 

1→**d**

Tips and Generalizations

These examples show how rather complex antenna and array equations can be better understood by making a few exploratory polar plots with the TI-89.

So far, only equations have been graphed. The next chapter shows that lab data also can be plotted.