Advanced Algebra Nomograph

ID: 8267

Time required
45 minutes

Activity Overview

*This activity is similar to the idea of a function machine. There are two levels of the manipulative (called a **nomograph**). The first is comprised of two vertical number lines, input on the left and output on the right. The second has three number lines to accommodate displaying the composition of two functions. At the first level, students try to find the rule of a hidden function by entering domain values and observing how they are transformed to new (range) values. The transformation is illustrated dynamically by an arrow that connects a domain entry to its range value. At the second level, students investigate composite functions. Inverse functions are treated as special cases of composition.*

Topic: Sequences, Series, & Functions

- *Calculate the value of a function f(x) defined by an algebraic expression at any real value of x.*

Teacher Preparation and Notes

This activity is appropriate for students in Algebra 2 or Precalculus.

- *Prerequisites are: an introduction to functions (including the terms domain and range), function notation ("y=" and "f(x)="), and experience graphing linear functions using slope and y-intercept. It is important that the model be demonstrated to students prior to them exploring the .tns file on their own. (Perhaps work through Problem 1 as a class.)*
- *This activity is designed to have students explore **individually and in pairs**. However, an alternate approach would be to use the activity in a whole-class format. By using the computer software and the questions found on the student worksheet, you can lead an interactive class discussion on functions and their inverses.*
- *Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.*
- ***To download the student TI-Nspire document (.tns file) and student worksheet, go to education.ti.com/exchange and enter "8267" in the keyword search box.***

Associated Materials

- *AdvAlgNomograph_Student*
- *AdvAlgNomograph.tns*

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

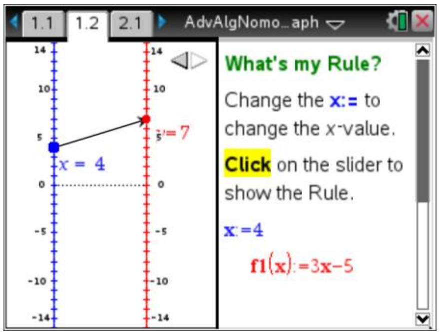
- *Algebra Nomograph (TI-Nspire Technology) — 8266*
- *What's my Rule? (TI-Nspire Technology) — 13293*
- *Radical Functions (TI-Nspire Technology) — 8978*

A ***nomograph*** is similar to a function machine in that it relates a number from one set (the domain) to a number in a second set (the range). Each set of numbers is represented in a pair of vertical number lines; the domain is on the left, and the range is on the right. According to the function rule, an element of the domain is mapped to its corresponding range element, and this mapping is depicted by an arrow.

Prior to beginning Problem 1, review domain and range, and ensure that students understand how to use the model.

Problem 1 – "What's my Rule?"

The first several problems on page 1.2 are "What's my Rule?" activities. Input values are entered, one at a time, into **x:=**. Pressing enter accepts the changes. The nomograph displays the input and its corresponding output. By repeatedly entering different inputs, the student should be able to discover the function's rule.



For example, if domain values 1, 2, 5, and 7 and their respective range values 3, 5, 11, and 15 are observed, the rule $f(x) = 2x + 1$ should be identified. When students have conjectured a rule, they should record it on their worksheets and check it. The rule is checked by selecting an input number, applying the rule, and predicting the output number.

Students can display the rule by clicking on the slider.

Solution

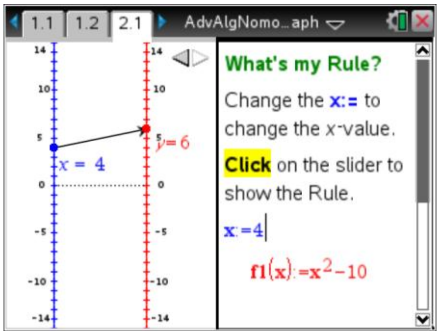
- For the given $x = 4$, **f1(*x*) = 3*x* – 5**

TI-Nspire Navigator Opportunity: *Live Presenter*

See Note 1 at the end of this lesson.

Problem 2 – A more difficult "What's my Rule?"

This nomograph on page 2.1 follows a quadratic rule. Students are guided through the same steps to determine the rule. Encourage students to record several of the ordered pairs they observed on their worksheets. This will help them in determining the function's rule.

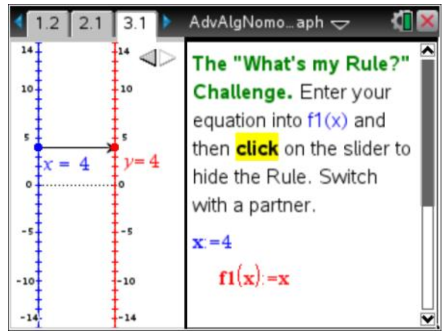


Solution

- For the given $x = 4$, **f1(*x*) = x^2 – 10**

Problem 3 – The "What's my Rule?" Challenge

On page 3.1, instruct students to create their own functions of the form $y = ax + b$ or $y = ax^2 + b$ (where a and b are integers). Each student should change **f1(x):=**, press enter, and then click the slider to hide the equations. Students should then exchange handhelds with a partner. It is the partner's task to use the nomograph to identify the mystery function. Encourage students to repeat this activity several times.



Solutions

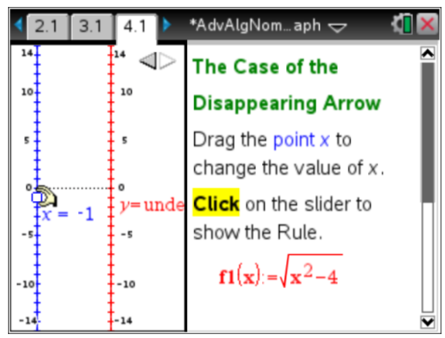
- Functions will vary.

TI-Nspire Navigator Opportunity: *Class Capture*

See Note 2 at the end of this lesson.

Problem 4 – The case of the disappearing arrow

The nomograph on page 4.1 displays a function with restricted domain: $\mathbf{f1}(x) = \sqrt{x^2 - 4}$. It is also a continuous nomograph—the inputs are changed by grabbing and dragging the point on the domain. As the point is dragged through x-values not in the domain, the function arrow between x and y disappears. Students are asked to explain when and why this happens for this specific function. To avoid confusion, make sure the arrow is visible when students first open the file (that is, $|x| > 2$).

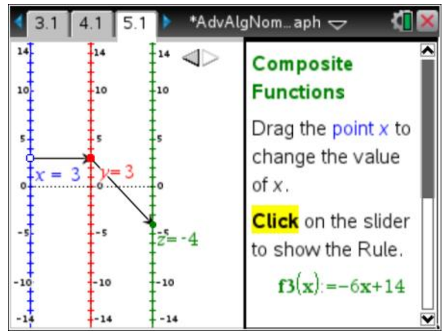


Solutions

- **–2 < *x* < 2**
- Possible answer: **The square root of a negative number is undefined.**

Problem 5 – Composite functions: "wired in series"

The nomograph on page 5.1 consists of three vertical number lines and behaves like *two* function machines wired in series. The point at x identifies a domain value on the first number line and is dynamically linked by the function $\mathbf{f1}(x) = 3x – 6$ to a range value y on the middle number line. That value is then linked by a second function $\mathbf{f2}(x) = –2x + 2$ to a value z on the far right number line.



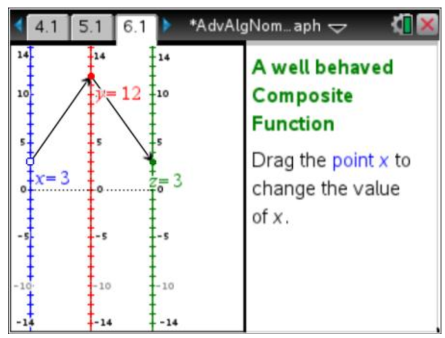
This nomograph enables students to explore the meaning of the composition of functions.

Be sure students are familiar with both notations for composite functions: **f2 ∘ f1** and **f2(f1(x))**. The input for the first function is controlled by grabbing and dragging the point at *x* and an arrow connects *x* to its output, *y*. The point *y* is used as input for a second function, and connected by a second arrow to its corresponding output, *z*.

Solutions

- **f3(*x*) = –6*x* + 14**
- **f2**(**f1**(**3**)) = **–4**; **f1**(**f2**(**3**)) = **–18**
- ***Yes*, the order matters.**

Problem 6 – A well-behaved composite function

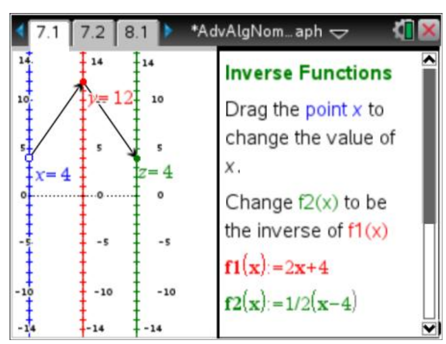


On page 6.1, the concept of an inverse function is introduced. The nomograph shows the function **f1**$(x) = 3x + 3$ and its mystery inverse **f2**, left for the student to determine. Here, they should find that **f1 ∘ f2** gives the same value as **f2 ∘ f1**. However, as they saw in Problem 5, this is not always the case. Have them consider an additional pair of functions **f1**(*x*) = *x* + 2 and **f2**(*x*) = x^2 to see that the order of composition does matter. (The order does not matter *if and only if* **f1** and **f2** are inverses.)

Solutions

- Possible answer: **The final output *z* is equal to the initial input value *x*.**
- **f2**$(x) = \frac{1}{3}(x-3)$
- **f2**(**f1**(**3**)) = **3**; **f1**(**f2**(**3**)) = **3**
- Possible answer: **Order does not matter. For each *x*, f1(f2(*x*)) = f2(f1(*x*)) = *x*.**

Problem 7 – Inverse functions



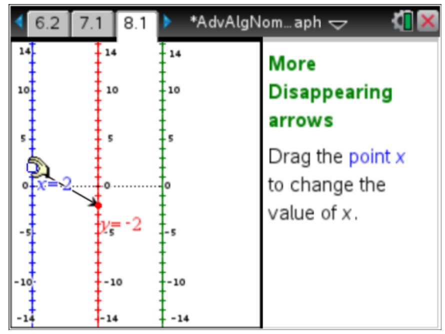
The formal definition of inverse functions is given here. You may wish to provide students with several (linear) functions and have them identify the functions' inverse. Encourage them to identify a function's inverse by switching *x* and *y* in the equation and solving for *y*. Also, you should reinforce that they need to find both *f*(*g*(*x*)) **AND** *g*(*f*(*x*)) to determine if two functions *f* and *g* are inverses. In Problem 9, students will explore some of these subtleties.

Solution **f2**$(x) = \frac{1}{2}(x-4)$

TI-Nspire Navigator Opportunity: *Quick Poll* and *Live Presenter*

See Note 3 at the end of this lesson.

Problem 8 – Missing arrows in a composition function



Domain restrictions on composite functions are examined. Arrows disappear when **f1**(x) = $2x – 6$ fails to be in the domain of **f2**$(x) = \sqrt{x}$.

Solutions

- **second arrow (for f2)**
- Possible answer: **It disappears when $2x – 6 < 0$ or $x < 3$ because the square root of a negative number is undefined.**

Problem 9 – "Almost" inverses and more disappearing arrows

On page 9.1, the composition of functions $f(x) = \sqrt{x}$ and $g(x) = x^2$ are compared. For $x > 0$, g appears to be the inverse of f, because $g(f(x)) = f(g(x)) = x$. But for $x < 0$, $g(f(x))$ is undefined because $f(x)$ is undefined. Both arrows disappear and thus f and g are not inverses.

TI-Nspire Navigator Opportunities

Note 1

Question 1, *Live Presenter*

Use Live Presenter to discuss the features of this problem. Show students how to enter new values for x and how the nomograph changes based upon the x-value or "input."

Note 2

Question 3, *Class Capture*

Use Class Capture to monitor student progress on question 3, offering help when needed. For additional practice, try sharing a student's screen with the entire class for any challenging functions they develop.

Note 3

Problem 7, *Quick Poll* and *Live Presenter*

You may choose to use Quick Poll to have students submit their responses for f2(x). For those students who enter a wrong result, it might be helpful to use Live Presenter to show the nomograph and have the class trouble shoot the response.