# Common Factors
## Teacher Notes

| TI-Nspire CAS | Investigation | Student | 40min |

Note – The TI Nspire CAS Prime Numbers activity would be useful to do before attempting this activity.

## Aim

The aim of this investigation is to learn different methods for finding the common factors of algebraic expressions.

National Curriculum Statement: Factorise algebraic expressions by identifying numerical factors (ACMNA191)
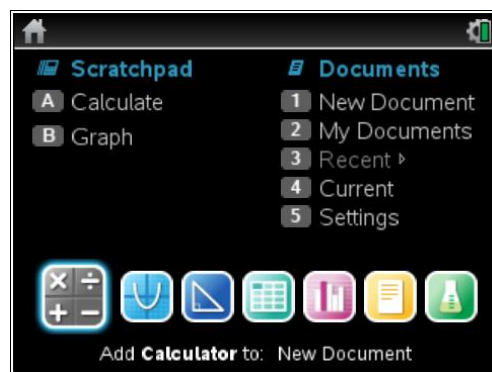
## Equipment

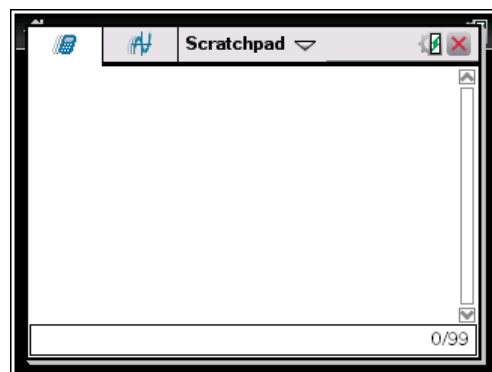For this activity you will need:

- TI-Nspire  (or TI-Nspire)

## Introduction – Setting up the calculations

For this activity, use a **Calculator** page or the **Scratchpad**.

To access a Calculator page, press **home** and select **Add Calculator**.

To access the Scratchpad, press **home** and select **Scratchpad** (or press **A**). Alternatively, press the ⌨ key (this key is not available on a Clickpad).

# Factors

A **factor** is whole number that divides evenly into another whole number. For example, the factors of 12 are 1, 2, 3, 4, 6 and 12 because there is no remainder when 12 is divided by one of these numbers.

# Common Factors

**Common factors** are factors that are common to two or more numbers. The factors of 12 and 18 are:

12 – **1**, **2**, **3**, 4, **6**, 12

18 – **1**, **2**, **3**, **6**, 9

The common factors are shown in boldface.

Therefore 1, 2, 3 and 6 are the **common factors** of 12 and 18.

# Highest Common Factor

The highest common factor (HCF) is the largest factor that is common to two or more numbers. From the list of common factors above, the HCF of 12 and 18 is **6**. The TI npire **Greatest Common Divisor** command will find the HCF of two numbers. The terms **greatest common divisor** and **highest common factor** have the same meaning.

Emphasise that the greatest common divisor and the highest common factor have the same meaning.

From the Scratchpad or a Calculator page, press **menu > Number > Greatest Common Divisor**.
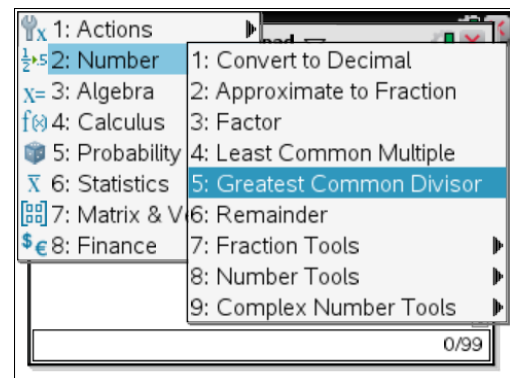


Once the **gcd** command appears on the screen, input two numbers separated by a comma. For this example, use **12,18**. The result confirms the highest common factor of **6** that we found previously.

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Use the **Greatest Common Divisor** command to find the highest common factor of the following pairs of numbers. The first entry has been completed for you.

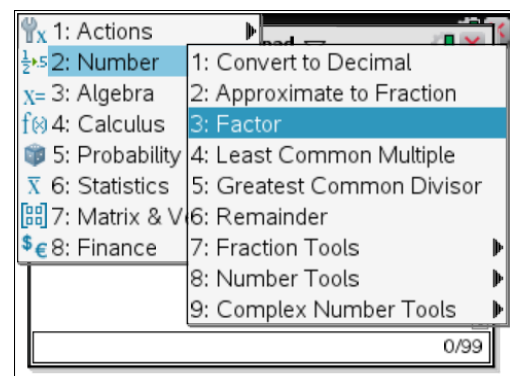| Numbers | Highest Common Factor |
|---------|----------------------|
| **12 & 18** | gcd(12,18) = 6 |
| **12 & 20** | gcd(12,20) = 4 |
| **30 & 45** | gcd(30,45) = 15 |
| **25 & 75** | gcd(25,75) = 25 |
| **48 & 72** | gcd(48,72) = 24 |
| **96 & 144** | gcd(96,144) = 48 |

## Prime Factors

Consider doing this prime factor approach before introducing the greatest common divisor command.

Another way to find the highest common factor is to use the **factor** command. This command gives you the prime factors of a number. Multiply the common prime factors together to find the highest common factor.

From the Scratchpad or a Calculator page, press **menu > Number > Factor**.

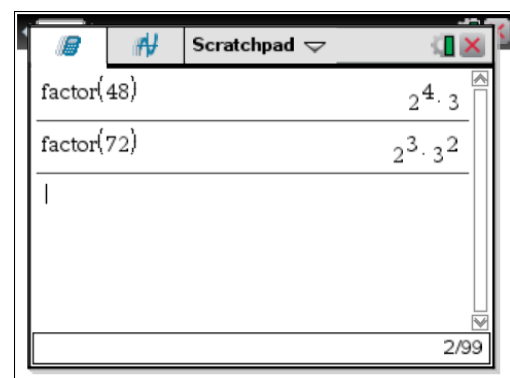(you can also press **menu > Algebra > Factor**)

The prime factors are:

$$48 = 2^4 \cdot 3 = \mathbf{2 \times 2 \times 2} \times 2 \times \mathbf{3}$$

$$72 = 2^3 \cdot 3^2 = \mathbf{2 \times 2 \times 2 \times 3} \times 3$$

The common prime factors are shown in boldface.

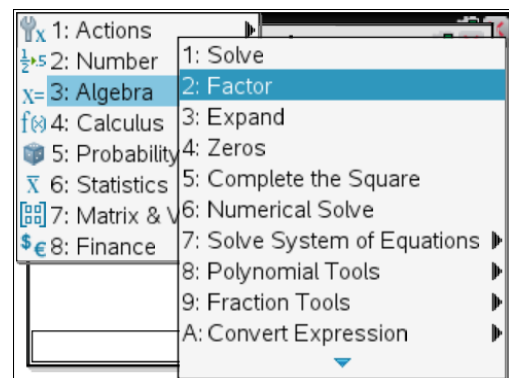Therefore the highest common factor is 2×2×2×3 = **24**.

This confirms the answer found previously.

Use the **factor** command to confirm the highest common factors that you found in the previous table.

| Numbers | Factors | HCF |
|---|---|---|
| **12 & 18** | 12 = **2**×2×**3**<br>18= **2**×**3**×3 | 2×3 = 6 |
| **12 & 20** | 12 = **2**×**2**×3<br>20 = **2**×**2**×5 | 2×2 = 4 |
| **30 & 45** | 30 =2×**3**×**5**<br>45 = **3**×3×**5** | 3×5 =15 |
| **25 & 75** | 25 = **5**×**5**<br>75 = 3×**5**×**5** | 5×5 = 25 |
| **48 & 72** | 48 = **2**×**2**×**2**×2×**3**<br>72 = **2**×**2**×**2**×**3**×3 | 2×2×2×3 = 24 |
| **96 & 144** | 96 = **2**×**2**×**2**×**2**×2×**3**<br>144 = **2**×**2**×**2**×**2**×**3**×3 | 2×2×2×2×3 = 48 |

# Common Factors in Algebra

When we factorise algebraic expressions, the first thing to look for is to see if there are any common factors. The TI Nspire factor command does this automatically. For example, to factorise $12x + 18$, do the following steps:
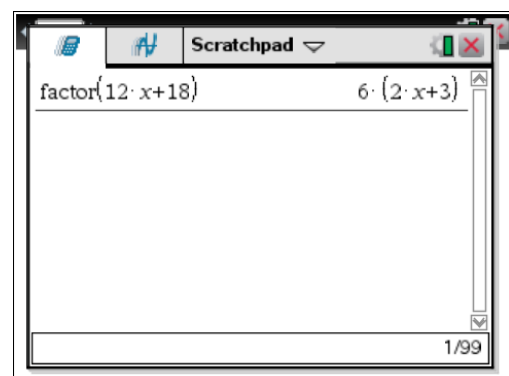
From the Scratchpad or a Calculator page, press **menu > Algebra > Factor**.



Input **12x+18** and press **enter**. The result is **6(2$x$+3)**. The highest common factor of 6 has been factorised out.

Confirm the answer by expanding it by hand or by using the **expand** command.

$$6(2x+3) = 6×2x + 6×3$$
$$= 12x + 18$$

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Using the Scratchpad or a Calculator page, factorise the following expressions using the **Factor** command. Check your answers by using the **Greatest Common Divisor** command to find the highest common factor of the numbers in the expression. The first entry has been completed for you.

| Expression | Factorise | Highest Common Factor |
|---|---|---|
| $8a+12$ | $4(2a+3)$ | gcd(8,12) = 4 |
| $18b+30$ | $6(3b+5)$ | gcd(18,30) = 6 |
| $75c-45$ | $15(5c-3)$ | gcd(75,45) = 15 |
| $12d+30$ | $6(2d+5)$ | gcd(12,30) = 6 |
| $35e-98$ | $7(5e-14)$ | gcd(35,98) = 7 |
| $200f+140$ | $20(10f+7)$ | gcd(200,140) = 20 |

## Least Common Multiple

The **least common multiple** (LCM) is a related command. The LCM is the lowest number that will divide evenly by two or more numbers. When adding and subtracting fractions, you often need to find the **lowest common denominator**, which is the same as finding the least common multiple.

From the Scratchpad or a Calculator page, press **menu > Number > Least Common Multiple**.
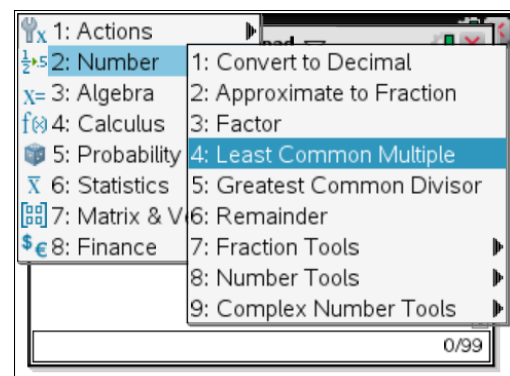


The least common multiple of 12 and 18 is **36**.
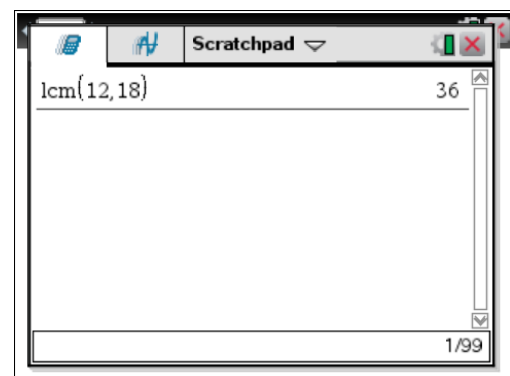
Use the **Least Common Multiple** command to find the least common multiple of the following pairs of numbers. The first entry has been completed for you.

| Numbers | Least Common Multiple |
|---------|----------------------|
| **12 & 18** | lcm(12,18) = 36 |
| **12 & 20** | lcm(12,20) = 60 |
| **30 & 45** | lcm(30,45) = 90 |
| **25 & 75** | lcm(25,75) = 150 |
| **48 & 72** | lcm(48,72) = 144 |
| **96 & 144** | lcm(96,144) = 288 |

**Extension Activity**

The following method can be difficult for some students. Encourage more capable students to attempt it.

You can also use the **factor** command to find the least common multiple. After you find the common prime factors, multiply them by the remaining prime factors that are not common. The factors of 48 and 72 are:

$$48 = 2^4 \cdot 3 = \mathbf{2 \times 2 \times 2} \times 2 \times \mathbf{3}$$

$$72 = 2^3 \cdot 3^2 = \mathbf{2 \times 2 \times 2 \times 3} \times 3$$

The common prime factors in boldface are **2×2×2×3**. The remaining prime factor of 48 is 2 and the remaining prime factor of 72 is 3. Therefore, the least common multiple will be **2×2×2×3**×2×3 = **144**. This verifies the previous answer.

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Use the **factor** command to confirm the least common multiples that you found in the table on the previous page.

| Numbers | Factors | LCM |
|---|---|---|
| **12 & 18** | 12 = **2**×2×**3**<br>18= **2**×**3**×3 | **2**×**3**×2×3 = 36 |
| **12 & 20** | 12 = **2**×**2**×3<br>20 = **2**×**2**×5 | **2**×**2**×3×5 = 60 |
| **30 & 45** | 30 =2×**3**×**5**<br>45 = **3**×3×**5** | **3**×**5**×2×3 = 90 |
| **25 & 75** | 25 = **5**×**5**<br>75 = 3×**5**×**5** | **5**×**5**×3 = 75 |
| **48 & 72** | 48 = **2**×**2**×**2**×2×**3**<br>72 = **2**×**2**×**2**×**3**×3 | **2**×**2**×**2**×**3**×2×3 = 144 |
| **96 & 144** | 96 = **2**×**2**×**2**×**2**×2×**3**<br>144 = **2**×**2**×**2**×**2**×**3**×3 | **2**×**2**×**2**×**2**×**3**×2×3 = 288 |

In this table, the common factors are written first in boldface followed by the factors that are not common. Encourage the students to write the factors from smallest to largest rather than in this order.