Extrema Using Derivatives

Extrema.tns

Name ______________________

Class ______________________

Problem 1 – Determine local extrema of $y = 2x^3 - 3x^2 - 12x$

Graph the function $y = 2x^3 - 3x^2 - 12x$. Use [–5, 5] for the x-dimensions and [–30, 30] for the y-dimensions.

1. How many local maximums do you see? Local minimums?
2. What does "point of inflection" mean?

Explore critical points and points of inflection on page 1.6 and 1.7. Notice the syntax for the **Solve** command. Find the first derivative of the function $y= 2x^3 - 3x^2 - 12x$. Set this function equal to zero and solve for x. Use the **Solve** command (**MENU > Algebra > Solve**).

3. What are the coordinates of the extrema? Explain which is the maximum and minimum.
4. Find the second derivative of the original function (or the derivative of the first derivative). Identify each critical points of the second derivative.
5. What is called if the second derivative is positive? Negative?
6. What is the point of inflection?
7. According to your graph, does the function change concavity there?
8. Use the **Analyze Graph** tool to verify the maximum and minimum. You will click to the left and right of the point of interest to restrict the domain. Did this verify the extrema you found using calculus?

Problem 2 – fMin and fMax

The **fMin** and **fMax** commands found in the Calculus menu are used to find the least and greatest y-values globally for the function. Since those values are $-\infty$ and $+\infty$ respectively, we have to restrict our domain for consideration.

For **fMin**, append **|x>0** to the command and for **fMax,** append **|x<0** to the command. How do these results compare with what you found above?

Problem 3 – The extrema of $y = x^3$

Take the first and second derivative of What are the critical points? Are there any extrema? If so, at what x-values? When does the function change concavity? What is the point of inflection? Show your work. Graph the function $f(x) = x^3$. Does the graph confirm the results you made using calculus.

Problem 4 – Extrema for other functions

Find the critical points for the following functions. Identify any extrema and points of inflection. Use calculus to justify your solutions.

Then graph the functions. Use **Analyze Graph** to verify the extrema. You can add a Calculator page and try using the commands **fMin** and **fMax** to make a second verification.

$g(x) = (x+1)^5 - 5x - 2$

- $g'(x) =$
- $g''(x) =$

$h(x) = \sin(3x)$

- $h'(x) =$
- $h''(x) =$

$j(x) = e^{4x}$

- $j'(x) =$
- $j''(x) =$

$k(x) = \dfrac{1}{x^2 - 9}$

- $k'(x) =$
- $k''(x) =$

Problem 5 – Evaluate endpoints and critical points

Many functions are defined on a limited domain. The roller coaster function is defined from 0 to 100 meters. Endpoints and critical points of a function should be evaluated. On page 5.2, change the profile of the roller coaster by using the arrows. Record the value of the endpoints and critical points. How does this information help you determine the extrema?

f3(0) =

f3(100)=