

Takeoff of an Airplane – ID: 8549

By Lars Jakobsson

Time required
45 minutes

Topic: Kinematics

- *Measure and describe one- and two-dimensional position, displacement, speed, velocity, and acceleration over time.*
- *Construct and interpret graphs of position, velocity, and acceleration versus time.*
- *Use appropriate equations to calculate average displacement, velocity, acceleration, and time in two dimensions.*

Activity Overview

In this activity, students will investigate the relationships between acceleration, velocity, and displacement. They will use acceleration data for a taxiing Boeing 737 to calculate the plane's velocity as it taxis down the runway. They will then use their calculated velocities to determine the plane's displacement during taxiing.

Materials

To complete this activity, each student will require the following:

- *TI-Nspire™ technology*
- *pen or pencil*
- *blank sheet of paper*

TI-Nspire Applications

Graphs & Geometry, Lists & Spreadsheet, Notes

Teacher Preparation

Many students have difficulty understanding the relationships between acceleration, velocity, and displacement. They especially struggle to translate graphs of these quantities vs. time into “real-life” scenarios. Using a well-known but infrequently modeled process—the takeoff of an airplane—allows them to apply these concepts to a familiar situation without risking the repetitiveness that can come with more everyday examples. You should review the basic kinematic equations with students and review simple examples to help them better follow the concepts in the activity.

- *The screenshots on pages 2–6 demonstrate expected student results. Refer to the screenshots on pages 7 and 8 for a preview of the student TI-Nspire document (.tns file).*
- ***To download the .tns file, go to education.ti.com/exchange and enter “8549” in the search box.***

Classroom Management

- *This activity is designed to be **teacher-led** with students following along on their handhelds. You may use the following pages to present the material to the class and encourage discussion. Note that the majority of the ideas and concepts are presented only in **this** document, so you should make sure to cover all the material necessary for students to comprehend the concepts.*
- *Students may answer the questions posed in the .tns file using the Notes application or on a blank paper.*
- *In some cases, these instructions are specific to those students using TI-Nspire handheld devices, but the activity can easily be done using TI-Nspire computer software.*

The following questions will guide student exploration in this activity:

- How are acceleration, velocity, and displacement related?
- How can we calculate velocity and displacement given only data on acceleration?

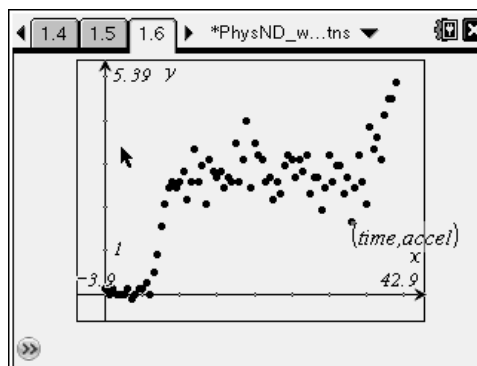
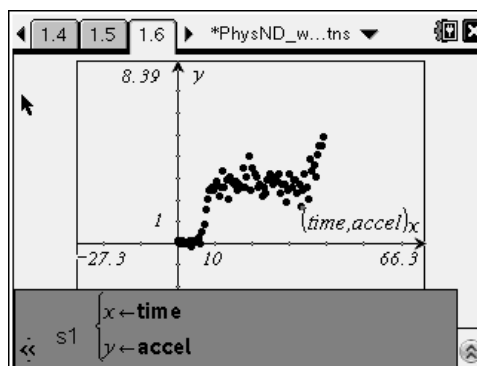
Students will carry out the activity with a pre-compiled data set. They will first calculate velocity using acceleration data. Then, they will calculate displacement data using velocity data.

Part 1 – Calculating velocity from acceleration data

Step 1: Students should open the file **PhysND_week02_airplane.tns** and read the first four pages. Page 1.5 contains the data students will use for their analyses. Before moving on to the next step, have students identify the time interval between data points (0.5 s) and describe the overall trend in the data (acceleration seems to increase with time).

	A	B	C	D
	time	accel		
1	0	0.128		
2	0.5	0		
3	1	0.128		
4	1.5	0		
5	2	0		

Step 2: Next, instruct the students to create a scatter plot of the data on page 1.6 (**Menu > Graph Type > Scatter Plot**). They should use **time** for the x-values and **accel** for the y-values. They should use the **Zoom-Fit (Menu > Window/Zoom > Zoom – Fit)** or **Window Settings (Menu > Window/Zoom > Window Settings)** function to resize the graph window so all the data are visible, but there is no extra space around the edge of the graph. Discuss the shape of the graph with the students. Students should then answer questions 1, 2, and 3 on pages 1.7 and 1.8. The questions and their answers (in italics) are given below.



TI-Nspire Navigator Opportunity: Screen Capture

Screen Capture can be used here to ensure that students have selected the correct variables for their scatter plots and resized the graph window appropriately. Be sure that students have correctly selected the variables **time** for the x-values and **accel** for the y-values. An appropriate window setting has an x-value range from -5 to 45 and a y-value range from -0.5 to 5.5. If student graphs look inappropriate, have them recheck the plotted variables and window settings.

- Q1.** What does the shape of the graph tell you about the motion of the airplane?
- A.** *The plane is not accelerating for the first few seconds. Then, its acceleration increases sharply before appearing to level off. This means the plane probably started from rest, and then began accelerating down the runway.*
- Q2.** Predict what a graph of velocity vs. time for the airplane would look like. Sketch it on a blank sheet of paper.
- A.** *Student predictions will vary. Encourage them to think about what a change in acceleration implies about a change in velocity. They should share and discuss their sketches before continuing with the activity.*
- Q3.** Predict what a graph of displacement vs. time for the airplane would look like. Sketch it on a blank sheet of paper.
- A.** *Student predictions will vary. Encourage them to think about what changes in acceleration and velocity imply about changes in displacement. They should share and discuss their sketches before continuing with the activity.*

Step 3: Next, students will use a formula to calculate the change in velocity for each time point. Remind students of the equation relating acceleration and velocity ($a = \frac{\Delta v}{\Delta t}$). Have them rearrange this equation to solve for Δv ($\Delta v = a \Delta t$). They should enter this equation into column B of the *Lists & Spreadsheet* application on page 1.9, as shown. (Remind them that the time interval, Δt , is the same—0.5—for each point. Therefore, the right-hand side of the rearranged equation becomes simply $0.5 \cdot \text{accel}$, as shown.) They should assign this column to the variable **dv** by typing **dv** into the title square (white box) of column B. Students can resize the columns (**Menu > Actions > Resize > Resize Column Width**) to see the expressions more easily.

A	B	C	D
time	dv		
	=0.5*accel		
1	0	0.064	
2	0.5	0.	
3	1	0.064	
4	1.5	0.	
5	2	0.	

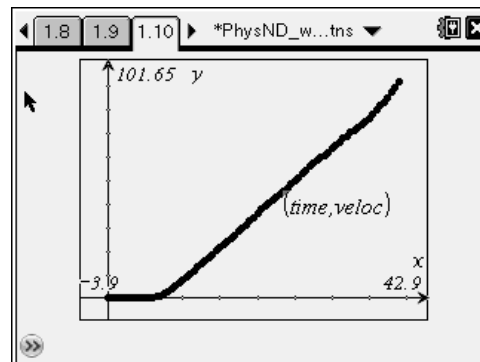
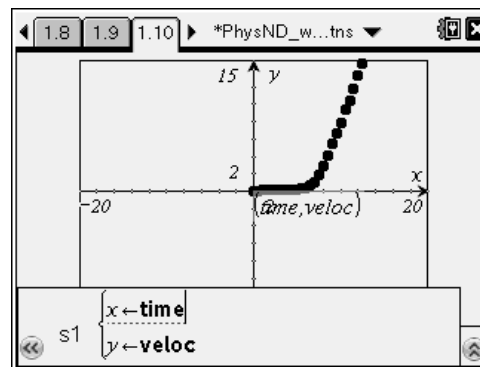
Formula bar: B dv:=0.5*accel

Step 4: Next, students must calculate the velocity of the plane at each time point. Explain that column B contains the change in velocity of the plane from each time point to the next. Ask students how they could use these data to determine the actual velocity of the plane at each point. Guide them to realize that adding consecutive changes in velocity will yield total velocity. They can use the **cumSum** function to find the cumulative sum of all the values in a column. Have students enter the expression **=cumSum(b)** in column C on page 1.9, as shown. (They can also select the **cumulativeSum()** function from the Catalog.) They should assign this column to the variable **veloc**.

A	B	C	D
time	dv	veloc	
	=0.5*accel	=cumulative	
1	0	0.064	0.064
2	0.5	0.	0.064
3	1	0.064	0.128
4	1.5	0.	0.128
5	2	0.	0.128

Formula bar: C veloc:=cumulativeSum(b[1])

Step 5: Next, students create a scatter plot of velocity vs. time for the plane on page 1.10. They should use **time** for their x -values and **veloc** for their y -values. Then, they will answer questions 4, 5, and 6 on pages 1.11 and 1.12. The questions and their answers (in italics) are given below



- Q4.** Does the graph of velocity vs. time agree with your prediction? If not, explain why you made the prediction you did. What assumptions did you make that were incorrect?
- A.** *Students' answers will vary. Encourage metacognitive thinking to help them better understand the relationship between acceleration and velocity.*
- Q5.** What does the graph of velocity vs. time tell you about the motion of the airplane?
- A.** *Initially, the plane has zero velocity, so it is at rest. Then, its velocity increases at a fairly constant rate.*

Q6. Do you still think the prediction for the graph of displacement vs. time that you made in question 3 is correct? If not, modify it based on your new thinking.

A. *Student predictions will vary. Encourage them to discuss why they did or did not change their graphs from question 3. Students should share and discuss their graphs before continuing with the activity.*

Part 2 – Calculating displacement from velocity data

Step 1: Next, students will use a formula to calculate the change in displacement for each time point. They should follow a procedure similar to that in step 3 of part 1, but they should substitute **veloc** for **accel** in the equation they enter in column B of the *Lists & Spreadsheet* application on page 1.13, as shown. They should assign this column to the variable **ds** by typing **ds** into the title square (white box) of column B. (Note: The handheld may run slowly after students enter the formula for **ds**. Students should wait until the clock icon disappears before continuing to enter commands.)

A	B	C	D
time	ds		
	=0.5*veloc		
1	0	0.032	
2	0.5	0.032	
3	1	0.064	
4	1.5	0.064	
5	2	0.064	

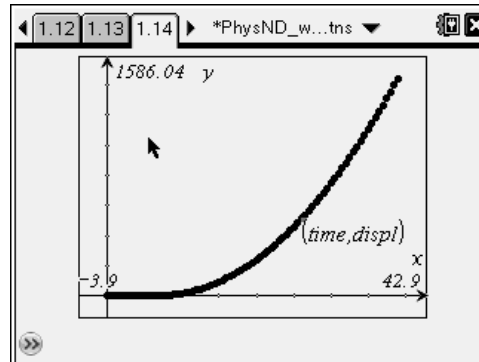
ds: =0.5*veloc

Step 2: Next, students will use the **cumSum** function to calculate the total displacement of the plane at each time point. They will use a similar procedure to that in step 4 of part 1. Have students enter the expression **=cumSum(b)** in column C on page 1.13, as shown. (They can also select the **cumulativeSum()** function from the Catalog.) They should assign this column to the variable **displ**.

A	B	C	D
time	ds	displ	
	=0.5*veloc	=cumulativeSum(B)	
1	0	0.032	0.032
2	0.5	0.032	0.064
3	1	0.064	0.128
4	1.5	0.064	0.192
5	2	0.064	0.256

displ: =cumulativeSum(B[[]])

Step 3: Next, students create a scatter plot of displacement vs. time for the plane on page 1.14. They should use **time** for their x-values and **displ** for their y-values. Then, they will answer questions 7, 8, and 9 on pages 1.15, 1.16, and 1.17. The questions and their answers (in italics) are given below.



- Q7.** Does the graph of displacement vs. time agree with your prediction? If not, explain why you made the prediction you did. What assumptions did you make that were incorrect?
- A.** *Students' answers will vary. Encourage metacognitive thinking to help them better understand the relationships between acceleration, velocity, and displacement.*
- Q8.** Do the graphs of acceleration, velocity, and displacement vs. time match what you would expect for an airplane taxiing for takeoff? Explain your answer.
- A.** *When a plane taxis for takeoff, it accelerates. Its velocity and displacement increase significantly over time. The graphs show this type of motion.*
- Q9.** Describe how the graphs of acceleration, velocity, and displacement vs. time would be different if the plane were taxiing from landing instead of taxiing for takeoff. Explain your answer.
- A.** *If the plane were landing instead of taking off, its velocity would be decreasing, and its acceleration would probably also be decreasing. Its displacement could be increasing or decreasing, depending on the reference point one used (i.e., the point where the plane touched down, or the point it is taxiing to). Therefore, a graph of acceleration or velocity vs. time would show a decreasing trend that ends at 0, and a graph of displacement vs. time would show either an increasing or a decreasing trend, depending on the reference point.*

Takeoff of an Airplane – ID: 8549

(Student)TI-Nspire File: *PhysND_week02_airplane.tns*

<p>TAKEOFF OF AN AIRPLANE</p> <p>Physics</p> <p>Kinematics</p>	<p>In this activity, you will explore the relationships between acceleration, velocity, and distance using acceleration data for the takeoff of a Boeing 737. The data were collected as the airplane taxied down the runway in preparation for takeoff.</p>	<p>Acceleration (a) and velocity (v) are related by the following equation:</p> $a = \frac{\Delta v}{\Delta t}$ <p>Similarly, velocity and displacement (s) are related by the following equation:</p> $v = \frac{\Delta s}{\Delta t}$
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<p>You will use the acceleration data for the Boeing 737 to calculate its velocity and displacement during takeoff.</p> <p>The spreadsheet on the next page contains time and acceleration data for the airplane. The units of time are seconds, and the units of acceleration are meters per second squared.</p>	<table border="1"> <thead> <tr> <th>A</th> <th>B</th> <th>C</th> <th>D</th> </tr> <tr> <th>time</th> <th>accel</th> <th></th> <th></th> </tr> </thead> <tbody> <tr><td>1</td><td>0</td><td>0.128</td><td></td></tr> <tr><td>2</td><td>0.5</td><td>0</td><td></td></tr> <tr><td>3</td><td>1</td><td>0.128</td><td></td></tr> <tr><td>4</td><td>1.5</td><td>0</td><td></td></tr> <tr><td>5</td><td>2</td><td>0</td><td></td></tr> </tbody> </table>	A	B	C	D	time	accel			1	0	0.128		2	0.5	0		3	1	0.128		4	1.5	0		5	2	0		
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1	0																															
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