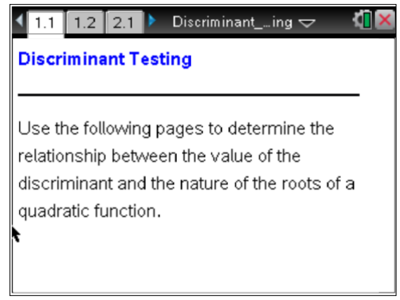



## Math Objectives

- Students will determine the relationship between the value of the discriminant and the nature of the roots of a quadratic function.
- Students will use the value of the discriminant to identify the number of roots of a quadratic function.
- Students will identify whether, based on the value of the discriminant, the graph of a quadratic function intersects the *x*-axis in zero, one, or two points.
- Students will determine the nature of the roots based upon the graph of a quadratic function.
- Students will look for and make use of structure. (CCSS Mathematical Practice)
- Students will use appropriate tools strategically. (CCSS Mathematical Practice)

## Vocabulary

- discriminant
- quadratic formula
- parameters
- nature of the roots

## About the Lesson

- This lesson involves discovering the relationship between the value of the discriminant and the nature of the roots of quadratic functions.
- As a result, students will:
  - Use a slider to generate graphs of random quadratic functions.
  - Make conclusions about the relationship between the value of the discriminant and the nature of the roots for the three cases.

## TI-Nspire™ Navigator™ System

- Use **Screen Capture** to examine students' randomly generated graphs.
- Use **Live Presenter** to monitor student progress and demonstrate the correct procedures.
- Use **Quick Poll** to assess students' understanding of the concepts throughout or after the lesson.

**TI-Nspire™ Technology Skills:**

- Download a TI-Nspire document
- Open a document
- Move between pages
- Use a slider to provide graphs of random quadratic functions

**Tech Tips:**

- To change slider values, click or press the up or down arrows.
- You can use the **Scratchpad** without adding pages to a document.

**Lesson Materials:**

*Student Activity*

Discriminant_Testing_Student.pdf

Discriminant_Testing_Student.doc

*TI-Nspire™ document*

Discriminant_Testing.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

## Discussion Points and Possible Answers

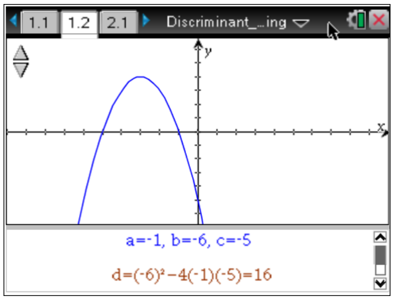
**Tech Tip:** Use the slider to generate a new random quadratic function. The parameters of the quadratic equation and the calculation of the value of the discriminant are shown.

**Teacher Tip:** Review with students that the discriminant is $b^2 - 4ac$, which comes from the quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where $a$, $b$, and $c$ are the parameters of the quadratic equation, $ax^2 + bx + c = 0$. Remind students that the value of the discriminant is important when determining the roots of a quadratic function. If desired, you may have students do several examples by hand to check the value of the discriminant or to algebraically find the roots of the equations of the functions generated.

**Teacher Tip:** As the investigation continues, you might ask students to solve the equation displayed on the screen and see how it relates to what is on the screen. You might also ask how their answers relate to the *x*-intercepts of the graphs.

**Tech Tip:** Another alternative is to have students solve a quadratic equation and describe the nature of the roots that the teacher software generates. This could be used in conjunction with TI-Navigator™ as **Quick Poll** questions.

**Move to page 1.2.**




How is the quadratic formula related to the graph of a quadratic function? What potential problems emerge in using the quadratic formula? The discriminant can help us with these issues.

1. Use the slider (click the up or down arrow) to produce graphs of quadratic functions. Notice the number of times the graphs intersect the *x*-axis. Describe the nature of the roots for the set of quadratic functions you can generate on page 1.2.

   **Answer:** The quadratic functions all cross the *x*-axis twice. The roots are two distinct real numbers.

2. a. Describe the value of the discriminant for all of the functions you can generate on page 1.2.

   **Answer:** The value of the discriminant is a positive number.

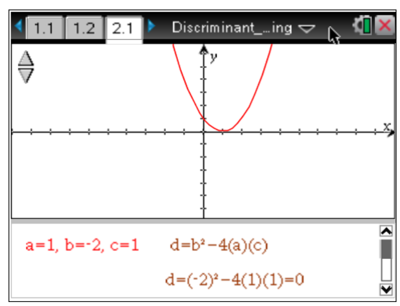
   > **Teacher Tip:** The quadratic functions are random, so the students will have various graphs. Students may work by clicking up or down on the slider. You may want to discuss why having a positive discriminant leads to having two real roots. Refer back to the quadratic formula and the ± square root of the discriminant.

   b. Explain how the value of the discriminant relates to your response to question 1.

   **Answer:** The discriminant is a positive number when the quadratic function has two distinct real roots. It is possible to find the square root of a positive number leading to two distinct answers when using the quadratic formula. The kinds of answers depend on whether or not the discriminant is a perfect square.

**TI-Nspire™ Navigator™ Opportunity: Screen Capture/Live Presenter**
**See Note 1 at the end of this lesson.**

**Move to page 2.1.**




3. Use the slider to produce various graphs of quadratic functions. Notice the number of times the graphs intersect the *x*-axis. Describe the nature of the roots for the set of quadratic functions you can generate on page 2.1.

   **Answer:** The quadratic functions all touch the *x*-axis once. The roots are two equal real numbers.

   > **Teacher Tip:** Students may not realize that the graphs in question 2 have two equal real roots. Since the graphs touch the *x*-axis only once, students may think that there is only one root. You may want to have a discussion to explain why having a zero discriminant leads to having two equal real roots. Refer back to the quadratic formula and the ± square root of zero. Also, this is called a double root in some textbooks. This might help lay the foundation for multiplicity of roots.

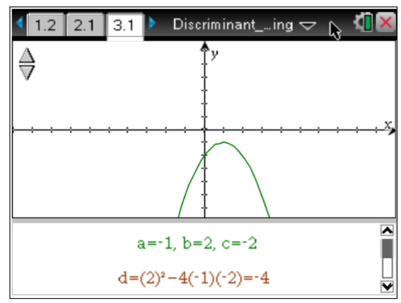
4. a. Describe the value of the discriminant for all of the functions you can generate on page 2.1.

   **Answer:** The value of the discriminant is always zero.

   b. Explain how the value of the discriminant relates to your response to question 3.

   **Answer:** The discriminant is zero when the quadratic function has two equal real roots. Since the square root of zero is zero, we add or subtract zero in the quadratic formula. This leads to two equal answers for the root.

**Move to page 3.1.**




5. Use the slider to produce various graphs of quadratic functions. Notice the number of times the graphs intersect the *x*-axis. Describe the nature of the roots for the set of quadratic functions you can generate on page 3.1.

   **Answer:** The quadratic functions do not intersect the *x*-axis. There are no real roots.

> **Teacher Tip:** Students may not realize that the graphs in question 3 have no real roots. Since the graphs never intersect the *x*-axis, students may not understand that there are two complex roots and no real roots. You may want to have a discussion to explain why having a negative discriminant leads to having no real roots. Refer back to the quadratic formula and the impossibility of taking the square root of a negative number in the real number system. This may lead to a discussion of complex numbers, if appropriate at this time.

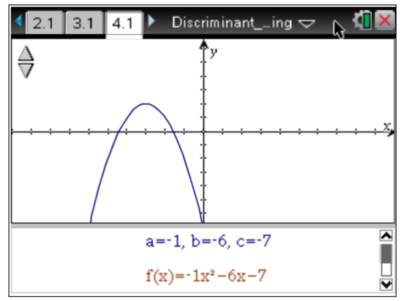
6. a. Describe the value of the discriminant for all of the functions you can generate on page 3.1.

   **Answer:** The value of the discriminant is a negative number.

b. Explain how the value of the discriminant relates to your response to question 5.

**Answer:** The discriminant is a negative number when the quadratic function has no real roots. In the quadratic formula you take the square root of the discriminant. This is not possible in real numbers when the discriminant is a negative number.

**Move to page 4.1.**




7. Test your conclusions with random quadratic functions by clicking the slider and using the **Scratchpad** to calculate the discriminant. Press [ ] on the TI-Nspire™ Touchpad or press [on] and choose **Calculate** on the left side under **Scratchpad**. Return to the document by pressing [esc]. Fill in the chart for 6 functions.

**Sample answers:** Students will generate random equations of graphic functions.

| *a* | *b* | *c* | Discriminant | Nature of the Roots | Root(s) |
|---|---|---|---|---|---|
| –1 | –8 | –17 | –4 | No real roots | |
| 1 | 0 | –2 | 8 | Two real roots | $\pm\sqrt{2}$ |
| –1 | 10 | –20 | 20 | Two real roots | $5\pm\sqrt{5}$ |
| –1 | 4 | –9 | –20 | No real roots | |
| 1 | –6 | 8 | 4 | Two real roots | 2 and 4 |
| –1 | –6 | –9 | 0 | Two equal real roots | –3 |

Use your chart and more examples to answer the following questions:

a. Why are there three possibilities for the number of *x*-intercepts (zero, one, or two intercepts) for all graphs? How is that determined?

**Answer:** The discriminant determines the three possible cases of *x*-intercepts (roots of the equation). This is because the value of the discriminant can be zero (one intercept), positive (two real roots or *x*-intercepts), or negative (no *x*-intercepts). Graphically, a parabola can cross the *x*-axis once, twice, or not at all.

b. When is the $b^2$ part of the discriminant negative?

**Answer:** Never. This is a common mistake made by students. Squaring a value always results in a positive number.

c. What determines whether you have real roots versus non-real roots?

**Answer:** If the discriminant is negative, the roots are not real.

d. How do the sizes of *a*, *b*, and *c* affect the discriminant?

**Answer:** The sizes of *a*, *b*, and *c* will determine the overall value of the discriminant. If $b^2 = 4ac$, the discriminant is zero and there is only one real root. If $b^2 > 4ac$, the discriminant is positive, so there are two real roots. If $b^2 < 4ac$, the discriminant is negative, so there are no real roots.

e. Will you ever have one real and one non-real root? Why or why not?

**Answer:** No. The discriminant cannot be both positive to give a real root and negative to give a non-real root. Graphically, if a parabola does not cross the *x*-axis, then the quadratic has two non-real (complex) roots. If a parabola crosses the *x*-axis twice or is tangent to it, both roots will be real because the *x*-axis is a real-number line.

f. If *a* and *c* are both negative, will there ever be two real roots? Why or why not?

**Answer:** When *a* and *c* are both negative, they will multiply to become a positive number. There will be two real roots if $b^2 > 4ac$, indicating that the discriminant is also a positive number.

g. When will the roots be rational? Explain mathematically.

**Answer:** When *a, b,* and *c* are rational and the discriminant is a perfect square, the value added to and subtracted from $-b$ will be an integer, and the roots will be rational.

h. When might the roots be integer values? Explain mathematically.

**Answer:** The roots will be integer values when two conditions are satisfied: First, the discriminant must be a perfect square so that there will be no rational numbers in the numerator of the quadratic formula $\left(-b \pm \sqrt{b^2 - 4ac}\right)$. Second, the denominator of the quadratic formula must divide equally into the numerators so that integer numbers are found $\left(\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right)$.

## Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- The relationship between the value of the discriminant and the nature of the roots of a quadratic equation for all three cases.
- The relationship between the value of the discriminant and the number of times the graph of the quadratic function intersects the *x*-axis.

## Assessment

You may use the Navigator system to assess the students' understanding of the lesson. Sample questions are below.

**Sample questions:**

1. The graph of the quadratic function has a minimum point on the *x*-axis. The value of the discriminant is:

   a) a positive number.
   b) zero.
   c) a negative number.

2. The graph of a quadratic function with no real roots:

   a) crosses the *x*-axis twice.
   b) intersects the *x*-axis in one point only.
   c) does not intersect the *x*-axis.

3. For a quadratic equation, the value of the discriminant is found to be 36. Without graphing, you can still describe the graph of the quadratic as intersecting the *x*-axis:

   a) twice.
   b) once.
   c) not at all.

4. For a quadratic equation, the value of the discriminant is found to be –49. The value of *a* in the equation $y = ax^2 + bx + c$ is also negative. Without graphing, you can still describe the graph of the quadratic as:

   a) opening upward with the vertex above the *x*-axis.
   b) opening upward with the vertex below the *x*-axis.
   c) opening downward with the vertex above the *x*-axis.
   d) opening downward with the vertex below the *x*-axis.

## TI-Nspire Navigator

**Note 1**

**Questions 1–6, Screen Capture/Live Presenter:** You may use the **Live Presenter** to demonstrate the procedures used in this document. You may also monitor the students' progress throughout the lesson. Students have random graphs, so you can use **Screen Capture** to show multiple graphs at once when discussing the discriminant and the nature of the roots of the quadratic functions.