Exploring the Equation of a Circle

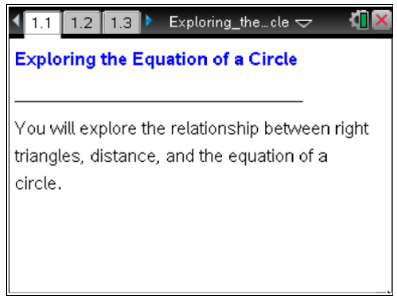
Student Activity

Name ______________

Class ______________

Open the TI-Nspire document *Exploring_the_Equation_of_a_Circle.tns.*

In this lesson, you will be able to visualize the definition of a circle and the relationship between the radius and the hypotenuse of a right triangle. By manipulating the size and location of different circles, you will see how the equation of a circle is derived.



Move to page 1.3.

1. Select **Menu > Trace > Geometry Trace**. Select point *P*. Then grab and drag point *P* to observe the path it traces.
   a. What do these points have in common?

   b. As you drag point *P*, a triangle moves along with the point. What changes about the triangle? What stays the same?

Move to page 1.4.

2. Read the definition of a circle given on this page. What does the word locus mean?

Move to page 1.5.

3. Drag point *P* around the circle. The equation of the circle and the coordinates of point *P* are given.
   a. What is the relationship between the hypotenuse of the right triangle and the radius of the circle?

   b. What is the relationship between the legs of the right triangle and point *P*?

   c. When given any right triangle and the lengths of its legs, what formula is used to find the length of its hypotenuse? Why is that helpful in this situation?

d. Since point *P* lies on the circle, what must be true about its coordinates? Pick a point and verify.

Move to page 1.6.

4. Change the radius of circle *O* by dragging point *Q* along the *x*-axis.
   a. When the radius of the circle changes, what changes in the equation? What stays the same?

   b. Why does the constant in the equation change?

Move to page 1.7.

5. Move the center of the circle away from the origin by dragging point *O*.
   a. How are the coordinates of the center of the circle related to the equation?

   b. What formula is used to find the length of radius $\overline{OP}$?

   c. Why is this formula similar to the equation of the circle?

Move to page 1.9.

6. Move the center of the circle by dragging point *O.* Change the radius of the circle by dragging point *Q*. Drag point *P* outside the circle, on the circle, and in the interior of the circle.
   a. What values are being substituted into the equation?

   b. Describe the location of point *P* when the inequality statement shows "**>**". Describe the location of point *P* when the inequality statement shows "**<**".

c. Drag point *P* until the statement becomes an equality (=). Where is point *P*?

d. Why do the constants within the parentheses in the equation and the coordinates of the center of the circle have opposite signs?

Move to page 1.11.

7. Move the center of the circle by dragging point *O.* Change the radius of the circle by dragging point *Q*. Drag point *P* around the circle. What do the *x*- and *y*-variables in the equation represent?

8. Suppose a circle has the equation $(x - 12)^2 + (y + 4)^2 = 25$.
   a. What is the radius of the circle?

   b. What are the coordinates of the center of the circle?

   c. How can you determine whether the point (12, –9) lies on the circle?

Move to page 2.1.

9. Follow the discussion led by your teacher for Problem 2 of this activity.

Move to page 3.1. After reading, move to page 3.2.

10. Select **Menu > Graph Entry/Edit > Equation > Circle >** $\boldsymbol{a \cdot x^2 + a \cdot y^2 + b \cdot x + c \cdot y + d = 0}$ to choose the standard form of a circle.
    - Type a 1 in the first box, then [tab] (a 1 will automatically be put in the second box), [tab]**,** then -4, [tab], 6, [tab] and finally -3.
    - A circle with center (2,-3) and radius 4 will be drawn.