

# Simulating randomness



## Teacher Notes and Answers

7 8 9 10 11 12



TI-Nspire



Investigation



Student



60 min

## Introduction

Randomness and chance affect events that have a major impact on our lives. In this task we explore methods for simulation randomness, and how the mathematics of chance can be used to analyse events affected by random factors.

## Part 1: Generating random numbers

Most software that generates random numbers uses a method that produces a decimal number between 0 and 1. To do this on the TI-Nspire CAS:

- Press **HOME-1** to create a new document, and then press **1** to add a **Calculator** page
- Press **MENU-6** to select Probability and then press **4** (Random) and then **1** (Number).
- Press **ENTER** to display a random number between 0 and 1.
- Press **ENTER** a few times to generate some extra random numbers.

You should see some random numbers generated. Note that you could also just type “**rand()**” and **ENTER** as well – whichever is quickest for you!

Note: When using the default display precision (FLOAT 6) only 6 decimal places are displayed on the right. However, if you use the random number in any calculations (see last line of screen shot), 15 decimal places of precision are used.



The **Random** command can be combined with the **Integer (int)** command to create random integers. Note that the **Integer** command returns only the whole number part of given number.

For example, the following shows how you might generate random numbers between 1 and 6.

- **rand()** will generate a random number between 0 and 0.9999 ... (inclusive).
- **6×rand()** will generate a random number between 0 and 5.9999 ... (inclusive).
- **6×rand()+1** will generate a random number between 1 and 6.9999 ... (inclusive).
- **int(6×rand()+1)** will generate a random integer between 1 and 6 (inclusive).

**Question 1.**

What command, using the **rand** and **int** functions, will generate a random integer between the following numbers? [Note: use your CAS to test if you wish.]

Many answers possible; sample answers are shown.

- a) 1 and 10 inclusive  
 $\text{int}(10 \times \text{rand}()+1)$
- b) 0 and 5 inclusive  
 $\text{int}(6 \times \text{rand}())$
- c) 3 and 5 inclusive  
 $\text{int}(3 \times \text{rand}()+3)$
- d) 11 and 20 inclusive  
 $\text{int}(10 \times \text{rand}()+11)$  or  $\text{int}(10 \times \text{rand}()+1)+10$
- e) –5 and 5 inclusive  
 $\text{int}(6 \times \text{rand}()) - \text{int}(6 \times \text{rand}())$

To make things a little more convenient, most calculators also have a random integer command that generates random integers between two limiting values. For example, the command **randint(11, 20)** on the TI-Nspire CAS (via **MENU-5-4-2**) will generate a random integer between 11 and 20 (inclusive).

**Question 2.**

Use your TI-Nspire CAS to check that the **randint** command works for each part in question 1.

- a)  $\text{randint}(1,10)$
- b)  $\text{randint}(0,6)$
- c)  $\text{randint}(3,5)$
- d)  $\text{randint}(11,20)$
- e)  $\text{randint}(-5,5)$

**Part 2: A simulation game**

For the remaining part of the exploration, use the **randint** command to generate the random integers needed. The random integer command can be used to simulate random events, or games that involve chance events.

For instance, the game Dice Cricket uses a die to simulate the innings (10 wickets) of each side in a simplified game of cricket. The result of each ball is simulated by the throw of a die, with the following results.

- If a 1 is rolled, then 1 run is scored.
- If a 2 is rolled, then 2 runs are scored.
- If a 3 is rolled, then 3 runs are scored.
- If a 4 is rolled, then 4 runs are scored.
- If a 5 is rolled, then the batsman is out (a wicket falls).
- If a 6 is rolled, then 6 runs are scored.

The die is rolled until each of the 10 batsmen is out.

**Question 3.**

What command using the **randint** function will simulate the results of each ball?

`randInt(1, 6)`

**Question 4.**

How often would you expect a wicket to fall? Explain your reasoning.

In the long run, we would expect a wicket to fall 1 in 6 balls. This is because a 5 is just as likely an outcome as any of the other numbers.

With a partner, play a game of Dice Cricket, using your answer to question c as the simulation command for each ball bowled. A scoring grid is attached.

When all the class has finished the game, answer the following questions.

**Question 5.**

List the innings scores for each team.

Answers will vary with the data generated by the students' innings scores.

**Question 6.**

Calculate the following statistics for the innings scores.

Answers will vary with the data generated by the students' innings scores.

- mean innings score
- lowest innings score
- highest innings score
- highest individual score

**Question 7.**

Comment on the range of scores obtained. (sample answers given only)

- What seems realistic about this simulation?  
The number of runs scored from each ball covers the common scores (i.e. 1, 2, 3, 4, 5 or 6). There is a chance element in each ball—a wicket could fall.
- What seems unrealistic about this simulation?  
Wickets will fall more quickly (on average) than in real cricket. It is common for first few batsman to score more runs (on average) before wicket falls. In real cricket, it is common for no runs to be scored for a given ball.

**Question 8.**

Suggest any improvements to make it more realistic.

Many answers possible. One obvious suggestion is to use more random integers (e.g. 10), to account for 0 runs, and for appeals (which could be decided by 50/50 chance using **randInt(0,1)**).

### Part 3: Randomness and sampling

If data is required from a very large population it is often time consuming and expensive to collect. An efficient sampling method called ‘regression to the mean’ can be used instead. It requires smaller random samples from the population and is therefore much cheaper and quicker to conduct. The following is an example of this method.

Let us assume that a test, marked out of 100, has 50 students sitting for it. The following scores were obtained.

45	18	2	78	32	90	61	87	19	62
50	17	25	92	16	25	47	22	64	31
54	73	29	16	5	12	43	72	19	55
27	68	81	93	7	54	61	70	61	33
2	30	44	88	17	52	34	91	77	64

First, enter the scores. To do this on the TI-Nspire CAS

- Press **HOME-1** to create a new document, and then press **3** to add a **Lists and Spreadsheet** page.
- At the top of column A, type the variable name ‘scores’.
- Enter the 50 test scores in column A.

The screen should appear as at right.



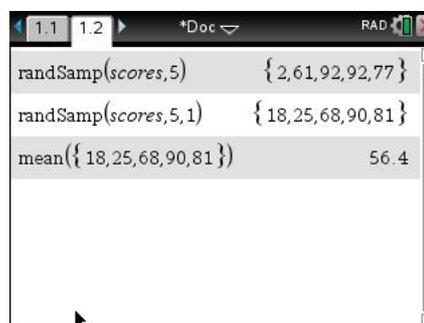
We have only a small amount of data, so it is not too difficult to enter; but to demonstrate the sampling method, we will select and use samples from this data.

The scores data can be sampled using the TI-Nspire CAS **randsamp** command. For instance, the command **randsamp(scores,5)** will sample 5 of the 50 scores. However, this will permit the random sampling to pick the same value more than once (see first line of sample screen overleaf).

We want to choose 5 scores with no repetitions. To do this, the **randsamp** command is changed to **randsamp(scores,5,1)** to indicate that repeat values are not permitted.

On your TI-Nspire CAS

- Press **HOME-1** to add a **Calculator** page.
- Press **MENU-5** to select **Probability**, and then press **4** (Random) and then press **5** (Sample).
- Type the following to complete the command **randsamp(scores,5,1)**.
- Type **mean(ans)** and then press **CTRL-ENTER** to calculate the mean of the five values as a decimal.



The five scores shown can be used as the first random sample. Note that the first sample of 5 scores have been done for you, and the mean of this sample has been calculated.

For the questions 9 to 14, show the results of your calculations in Table A.

**Question 9.**

i Use the **randsamp** function to generate another 5 pieces of data. This is your second sample. Find the mean of the second sample (and record its value in Table A). Is it close to the true mean?

Answers will vary with the data generated by the students' use of the **randsamp** command.

**Question 10.**

Average the means of the first and second samples, and record this value in Table A. Is the result better?

Answers will vary with the data generated by the students' use of the **randsamp** command.

**Question 11.**

Continue selecting samples and averaging the means of your samples. What do you notice?

The averaged mean should 'regress' or approach the actual mean after 7 or 8 samples.

**Question 12.**

How many samples did you need to get close to the true mean?

The averaged mean should 'regress' or approach the actual mean after 7 or 8 samples.

**Question 13.**

Repeat this process, this time using samples of 10. Record your results in Table B.

**Question 14.**

How did the size of the samples affect the number of trials needed for the average of the means of the samples to get close to the mean?

The larger sample size should result in a quicker regression to the mean.

### Dice cricket instructions

- Decide who will simulate the delivery of each ball, and who will record the results.
- When runs are scored, add them to the current run score and mark with an ‘x’ to indicate the updated score.
- When a wicket falls, circle the number of that wicket (also place a circle on the score at which the wicket fell).
- When 10 wickets have fallen, the innings is over. Swap roles.

### Dice cricket scoring grid

#### Team A

Wickets

1	2	3	4	5
6	7	8	9	10

Runs

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200
201	202	203	204	205	206	207	208	209	210
211	212	213	214	215	216	217	218	219	220
221	222	223	224	225	226	227	228	229	230
231	232	233	234	235	236	237	238	239	240
241	242	243	244	245	246	247	248	249	250
251	252	253	254	255	256	257	258	259	260
261	262	263	264	265	266	267	268	269	270
271	272	273	274	275	276	277	278	279	280
281	282	283	284	285	286	287	288	289	290
291	292	293	294	295	296	297	298	299	300

#### Team B

Wickets

1	2	3	4	5
6	7	8	9	10

Runs

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200
201	202	203	204	205	206	207	208	209	210
211	212	213	214	215	216	217	218	219	220
221	222	223	224	225	226	227	228	229	230
231	232	233	234	235	236	237	238	239	240
241	242	243	244	245	246	247	248	249	250
251	252	253	254	255	256	257	258	259	260
261	262	263	264	265	266	267	268	269	270
271	272	273	274	275	276	277	278	279	280
281	282	283	284	285	286	287	288	289	290
291	292	293	294	295	296	297	298	299	300

## Randomness and sampling tables

Table A: Samples of 5 scores

Sample No.	Mean	Total of means	Divide by	Mean of means
1	56.4	56.4	1	56.4
2	_____	$56.4 + \text{_____} = \text{_____}$	2	$(56.4 + \text{_____})/2 = \text{_____}$
3			3	
4			4	
5			5	
6			6	
7			7	
8			8	
9			9	
10			10	

Table B: Samples of 10 scores

Sample No.	Mean	Total of means	Divide by	Mean of means
1			1	
2			2	
3			3	
4			4	
5			5	
6			6	
7			7	
8			8	
9			9	
10			10	

## Teacher notes

- This task is designed to familiarise students with random number generators and some of their uses.
- The first part introduces the basic random number function, and how it can be modified to generate random numbers within a specified range. Once students have mastered its use, it seems reasonable to allow them to use the random integer function from then on.
- Most software, including the TI-Nspire CAS, use pseudo-random numbers, which sample from a number sequence known to produce consecutive values with digits that have no apparent order. For this reason, it is sometimes necessary to 'seed' the random number generator, which permits the user to start from a different position in the sequence (e.g. **RandSeed 98763524**).
- Part 2 allows students to experience the effects of randomness for the cricket simulation in a fun (and competitive) manner. Although we expect a wicket every 6 balls in the long term, it is possible to have long runs without a wicket, or a number of wickets in a row. Encourage students to reflect upon the variability of the results obtained.
- Part 3 focuses on the use of small random samples for estimating the mean, rather than obtaining results from the entire population. Voting analysis is one obvious application of this method. The effectiveness of this method relies on the random nature of the samples.
- An extension to Part 3 of this exploration is the use of the sampling method to survey the school (or year level) for some numerical variable. Each student could sample 10 students at random, and then average the means obtained.