Area and Perimeter of Regular Polygons -- An Extension

Teacher Note

Since the following exploration can be time consuming, you may wish to divide the polygons in Part **c** among several groups, then ask students to compile their results. (If so, all students should use the same radius when constructing the inscribed polygons.)

Exploration

a–b. Students use a geometry utility to draw an inscribed regular triangle, then determine its perimeter and area.

c–d. The following sample data was collected using a radius of 5 cm.

Regular Polygon	Perimeter (cm)	Area (cm^2)
triangle	26.0	32.4
square	28.3	49.9
pentagon	29.4	59.3
hexagon	30.0	64.8
octagon	30.6	70.6
18-gon	31.2	76.8
circle	31.4	78.4

Discussion

a. Sample response: The number of sides in the polygon equals its number of central angles. Since there are 360° in a circle, 360° divided by the number of sides (or vertices) determines how many degrees each central angle must have. To find the vertices, locate the points on the circle where each central angle has the desired measure.

b. The congruent central angles form congruent isosceles triangles in each polygon, with the base of each triangle representing a side of the polygon. Since the bases of these triangles are all congruent, the sides of the polygon are congruent. Since the base angles of all the isosceles triangles are congruent, the angles of the polygon are congruent.

c.

1. One method divides the regular polygon into congruent isosceles triangles with vertex angles at the center of the polygon and bases as the sides of the polygon. The area of each of these triangles, where a is the height and b is the length of the base, is

   $$\frac{1}{2}ab$$

   The area of the polygon is equal to the sum of the areas of these triangles. If the polygon has n sides, then the area of the polygon is

   $$n\left(\frac{1}{2}ab\right)$$

   Since $n \bullet b$ is the perimeter of the polygon, the generalized form of this method is the formula

   $$A = \frac{1}{2}ap$$

   where a is the apothem and p is the perimeter. **Note:** Students also may suggest using technology, as described in the exploration.
2. Sample response: Each method can be useful, depending on the information given. Using a formula is appropriate when you know the required lengths. Using technology can be faster if you have already drawn a model of the polygon.

d.

1. The shape of the polygon approaches a circle.
2. The area of the polygon approaches the area of the circle.
3. The perimeter of the polygon approaches the circumference of the circle.

e.

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2. The area of the polygon approaches the area of the circle.
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f.

1. Sample response: As the number of sides of the polygon increases, the shape of the polygon becomes more and more circular. The apothem approaches a radius (r) of the circle and the perimeter approaches the circumference ($2\pi r$) of the circle.

2. As the number of sides increases, the formula for the area of a regular polygon approaches the formula for the area of a circle:

$$A = \frac{1}{2}r(2\pi r) = \pi r^2$$