The Gradient Function

Answers

Student Activity

7 8 9 10 **11** **12**

TI-Nspire Investigation Student 30 min

Introduction

The gradient of a straight line can be calculated using a formula, but what about the gradient of a curve? The gradient of a curve is not constant. This investigation introduces the notion of the *gradient function*, a set of points that describe the gradient of a curve.

Key Terms

Tangent A tangent is a line that shares the same **gradient** of the curve at that point.

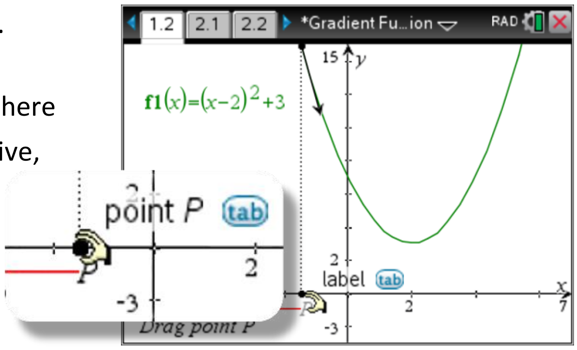
Gradient In the context of the Cartesian plane the gradient is the slope of the curve.[1]

Sign Table This is similar to a 'table of values' for a function, however it is only the sign (+ / -) that is required for each corresponding value of x.

Looking for a Sign

Open the TI-nspire document and navigate to page 1.2.

The point P can be moved along the x axis to explore where the gradient of the function $f(x)=(x-2)^2+3$ is positive, negative or zero.



To grab point P, move the mouse over P. Objects in the vicinity may become active, use the TAB key to toggle between local objects until 'point P' is displayed. Press and hold the centre of the touch pad or press [Ctrl] then click. When point P is selected the hand closes around it, moving the mouse will now drag point P along the x axis. Press [Esc] to release the grip.

The first part of this investigation explores the gradient of the curve as either positive, negative or zero. As point P is moved along the x axis, three representations of the gradient are provided:

- The tangent visually displays the gradient of the curve.
- The words: positive, negative or zero appear.
- A **sign** graph displays a dynamic summary of the information

[1] Slope or Gradient – When dealing with the Cartesian plane slope and gradient may be used synonymously, the word 'steepness' is different. A hill may be described as 'steep', however its common use does not imply any direction, positive or negative, so caution should be applied when using the word 'steep'

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The tangent line helps visualise information about the gradient of the curve, its linear representation allows us to use prior knowledge and understanding of the gradient to help us determine if the gradient is positive, negative or zero. The words positive, negative or zero reinforce this information and the sign graph provides a dynamic and on-going history of the same information. Use these representations to complete the following sign tables; use either '+' , '–' or '0' accordingly.

Question: 1.

Complete a sign table for the **gradient** of the function: $f(x)=(x-2)^2+3$.

x	−4	−3	−2	−1	0	1	2	3	4
Signed gradient of $f(x)$	–	–	–	–	–	–	0	+	+

Question: 2.

Navigate to page 2.1 and complete a sign table for the **gradient** of the function:

$$f(x)=\frac{(x-3)(x-1)(x+4)}{2}.$$

x	−4	−3	−2	−1	0	1	2	3	4
Signed gradient of $f(x)$	+	+	–	–	–	–	–	+	+

Question: 3.

On a piece of paper, graph the function $g(x)=\left(x^2-9\right)\left(x^2-1\right)$ and use either a pencil or ruler to represent the tangent line. Complete the sign table below for the gradient of the graph $g(x)$

Note: A graph application is located on page 2.3 to graph $g(x)$ if required.

x	−4	−3	−2	−1	0	1	2	3	4
Signed gradient of $g(x)$	–	–	+	+	0	–	–	+	+

Question: 4.

On a piece of paper, graph the function $p(x)=(3-x)(x+1)$ and use either a pencil or ruler to represent the tangent line. Complete the sign table below for the gradient of the graph $p(x)$.

x	−4	−3	−2	−1	0	1	2	3	4
Signed gradient of $p(x)$	+	+	+	+	+	0	–	–	–

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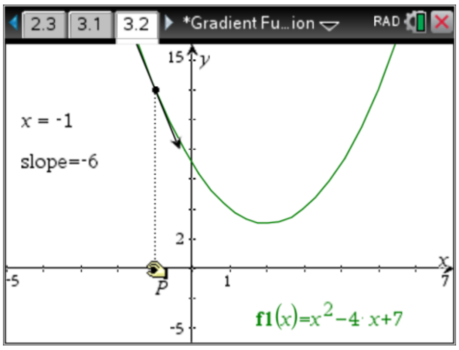
Gradient of a Quadratic

Navigate to page 3.2.

Point P can be moved along the x axis to explore the gradient of the function: $f(x) = x^2 - 4x + 7$. In this case the gradient or slope is quantified.



Question: 5.

a) Move point P along the line and use it to record the gradient of the function at each point.

x	−4	−3	−2	−1	0	1	2	3	4
Gradient of $f(x)$	**−12**	**−10**	**−8**	**−6**	**−4**	**−2**	**0**	**2**	**4**

b) Use the information (table) above to determine a rule for the gradient of $f(x)$.

Gradient function = $2x - 4$

Question: 6.

a) Navigate to page 4.1, move point P along the line and use it to record the gradient of $f(x) = x^2 - 2x - 6$ at each point.

x	−4	−3	−2	−1	0	1	2	3	4
Gradient of $f(x)$	**−10**	**−8**	**−6**	**−4**	**−2**	**0**	**2**	**4**	**6**

b) Use the information (table) above to determine a rule for the gradient of $f(x)$.

Gradient function = $2x - 2$

Question: 7.

Navigate to page 5.1. In this example the table of values is automatically generated and plotted as a graph. The quadratic function is: $f(x) = x^2 + x - 6$. Determine the equation to the line produced when the gradient is plotted. Gradient function = $2x + 1$

Note: The equation can be checked by typing it into the equation entry line.

Question: 8.

Navigate to page 6.1. This is another example where the table of values is automatically generated and plotted as a graph. The quadratic function is a little different this time: $f(x) = -x^2 + x + 6$. Determine the equation to the line produced when the gradient is plotted.

Gradient function = $-2x + 1$

Question: 9.

Navigate to page 7.1. The quadratic graph is dynamic, the gradient function responds accordingly. Manipulate the quadratic graph; observe and comment on the visual and algebraic changes to the quadratic and gradient function.

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There are many observations students can make in this section:

- Gradient function for a quadratic is always linear.
- Gradient function crosses x axis in line with turning point of quadratic.
  [*... gradient equals zero at turning point.*]
- Translation parallel to x axis of the original function also translates gradient function in the same way and by the same amount.
  [*...useful observation with regards to later use and understanding of types of problems involving the the chain rule.*]
- Translation parallel to y axis of the original function does NOT affect gradient function.
  [*...because the gradient with respect to x is unchanged.*]
- Students may establish part of the general rule with regards to:
  - Constant term in original function does not affect gradient function;
  - Coefficient of x becomes 'constant' in gradient function;
  - The x^2 in the original function becomes $2x$ in the gradient function.

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