Maclaurin Polynomials

ID: 10129

Time required
45 minutes

Activity Overview

In this activity, students will use TI-Nspire technology to explore Maclaurin polynomials. They will develop polynomials that approximate very special functions.

Topic: Maclaurin Polynomials

- *Compute the Maclaurin series for sin(x), cos(x), f(x) = e^x, and* $f(x)=\dfrac{1}{1-x}$.

Teacher Preparation and Notes

- *This investigation offers opportunities for students to begin their studies of power series approximations, deriving Maclaurin and Taylor series expansions for common functions.*
- *Students will use TI-Nspire technology to explore the Maclaurin polynomials. They produce polynomials that approximate sin(x), cos(x), and e^x. The Maclaurin polynomial will approximate a function over a small, specific interval.*
- *Students should already be familiar with a Taylor polynomial, that is:*
  $$P_n(x)=f(c)+f'(c)(x-c)+\frac{f''(c)}{2!}(x-c)^2+\ldots+\frac{f^{(n)}(c)}{n!}(x-c)^n$$
- *Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.*
- ***To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "10129" in the keyword search box.***

Associated Materials

- *MclaurinPolynomials_Student.doc*
- *MclaurinPolynomials.tns*
- *MclaurinPolynomials_Soln.tns*

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Taylor Polynomials (TI-84 Plus family) — 4375*
- *Taylor Polynomial Examples (TI-Nspire technology) — 16118*
- *Mr. Taylor, I Presume? (TI-Nspire technology) — 10096*

Functions that are continuous and differentiable at a center can be approximated by polynomials. One such way of doing this is to generate a Maclaurin polynomial for a function.

An approximating polynomial is to be expanded about the center *c* which is in the domain of a function *f*. If this *c* has the same value in a polynomial *P* and function *f* then *P*(*c*) = *f*(*c*). Graphically, *P*(*c*) = *f*(*c*) means that the graph of *P* passes through the point (*c*, *f*(*c*)).

A Maclaurin polynomial is a polynomial that is based upon a function's derivatives at *c* = 0. Specifically, the *n*th Maclaurin polynomial is defined as

$$P_n(x) = \frac{f(0)}{0!}x^0 + \frac{f'(0)}{1!}x^1 + \frac{f''(0)}{2!}x^2 + \ldots + \frac{f^{(n)}(0)}{n!}x^n$$

$$P_n(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \ldots + \frac{f^{(n)}(0)}{n!}x^n$$

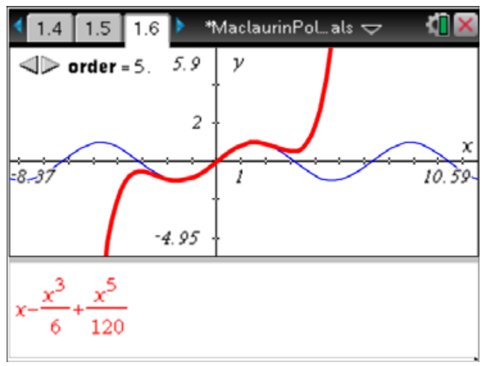
Problem 1 – Maclaurin polynomial for *f*(*x*) = sin(*x*)

In generating the third degree Maclaurin polynomial for *f*(*x*) = sin(*x*), we compute (Have students compute and fill the missing answers.)

$$P_3(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}(x)^2 + \frac{f'''(0)}{3!}(x)^3$$

f(0) = 0 *f''(0)* = 0

f'(0) = 1 *f'''(0)* = –1



Students can use CAS technology in the *Calculator* application to compute the derivative of sin(*x*) at a point by inserting a *Calculator* page (⌂on **> Calculator**) and pressing menu **> Calculus > Derivative at a Point**.

If CAS technology is not available, students can compute the derivative by hand and substitute the values into the Maclaurin polynomial. This results in

$$P_3(x) = 0 + (1)x + (0)x^2 - \frac{(1)}{3!}x^3$$

$$P_3(x) = x - \frac{1}{6}x^3$$

Have students study the graphs of *f*(*x*) and $P_n(x)$ on page 1.6. It is important to point to students that the polynomial begins its approximation from (0, *f*(0)). Students can change the order of the Maclaurin polynomial by clicking on the order arrows.

Q: What do you notice when the degree is 1 and 2. Why?

A: Their graphs are the same because the polynomials P_1 and P_2 are equal.

Q: What do you notice when the degree is 3 and 4. Why?

A: Their graphs are the same because the polynomials P_3 and P_4 are equal.

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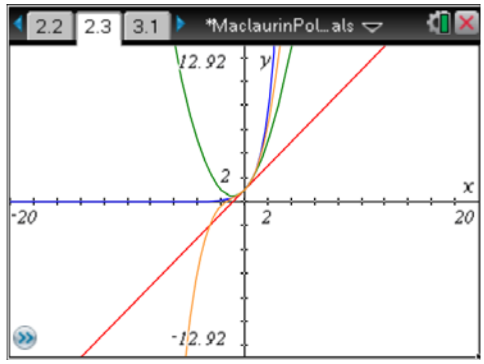
See Note 1 at the end of this lesson.

Problem 2 – Maclaurin polynomial for $f(x) = e^x$



1. Write $P_1(x)$, $P_2(x)$, and $P_3(x)$ for $f(x) = e^x$

$$P_1(x) = 1 + x$$

$$P_2(x) = 1 + x + \frac{x^2}{2!}$$

$$P_3(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$$

2. Graph $f(x)$, $P_1(x)$, $P_2(x)$, and $P_3(x)$

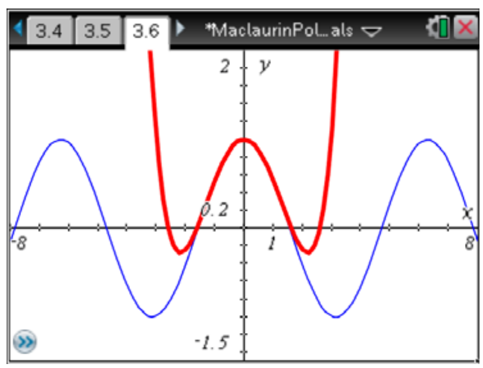
What do you notice?

All the approximations of e^x and the function $f(x)$ all have the value 1 at $x = 0$.

TI-Nspire Navigator Opportunity: *Live Presenter*

See Note 2 at the end of this lesson.

Problem 3 – Maclaurin polynomial for $f(x) = \cos(x)$



1. Find $P_8(x)$ for $f(x) = \cos(x)$

$$P_8(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!}$$

2. What do you notice about the value of each derivative after 0 has been substituted?

The cosine values are either 1 or –1 and the sine values are 0.

3. What do you notice about the approximated polynomial?

Every odd or other derivative is missing. For example, $f'(x)$, $f'''(x)$, $f^{(5)}(x)$, and $f^{(7)}(x)$

4. Write two expressions to describe you findings in the previous question, when differentiating $\cos(x)$, in terms of n.

$$f^{(2n)}(0) = (-1)^n$$

$$f^{(2n+1)}(0) = 0$$

5. Graph $P_8(x)$ and $f(x) = \cos(x)$. What do you notice?

The graph begins to take the shape of the cosine then goes to infinity.

TI-Nspire Navigator Opportunities

Note 1
Problem 1, *Live Presenter*
This would be a good place reinforce with students the difference between order and degree of a Maclaurin Polynomial. This can easily be done by clicking the order arrows and observe the changes in the polynomial at the bottom of the screen

Note 2
Problem 2, *Live Presenter*
You may want to go back to page 1.6, press [tab] and change *f1*(*x*) to e^x. In this way, you can continue to increase the order of the Maclaurin polynomial to illustrate what the questions in this problem are asking.