Texas Instruments Activity #13
Title: The Growth Rings of a Tree
Author: Charles P. Kost II
Estimated Time: 40-50 Minutes

NCTM Standards:

Connections Standard – Recognize and apply mathematics in contexts outside of mathematics. Use representations to model and interpret physical, social and mathematical phenomena.

Problem Solving Standard – Solve problems that arise in mathematics and other contexts.

Algebra Standard – Understand patterns, relations, and functions. Approximate and interpret rates of change from graphical and numerical data. Understand and compare the properties of classes of functions.

Topics in Calculus:

Derivatives, Limits, Definition of Derivatives, Applications of Derivatives

Overview:

In this activity, the students will use the limits to find derivatives and to find the relationship between radii and area. This activity uses tree growth as the basis of the activity.

Teacher Directions:

If possible, bring in a sample of a cross section of a tree. In most cases, the trees are not exact circles, however, there are cases where this occurs. Start the activity by showing the students the cross section and explaining the first paragraph of the activity in detail.

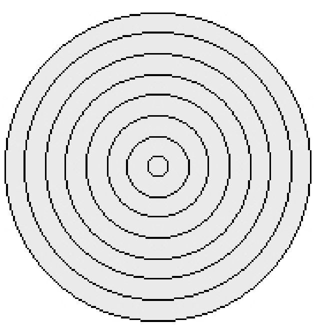
Supplies: TI-89 Graphing Calculator, Sample Tree Cross Section(s)

Name: ____ Date: _________________________

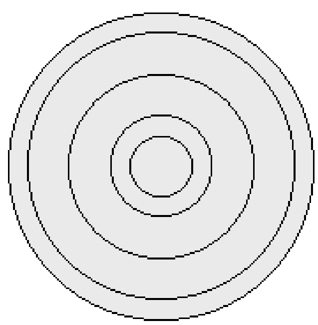
THE GROWTH RINGS OF A TREE

Each year, trees grow throughout the spring and summer months. In the winter, the trees produce a dark productive layer directly underneath the bark. This layer protects the new ring of tree that grows that year. However, it also provides a distinction between the layers so that we can count the rings and determine the age of the tree and the amount of growth the tree encountered during each year. In order to view these rings, the trees have to be cut horizontal with the ground.

1. Below are two models of tree growth. Which of the two models is the best depiction of real world tree growth? Why are the models not good models of tree growth?

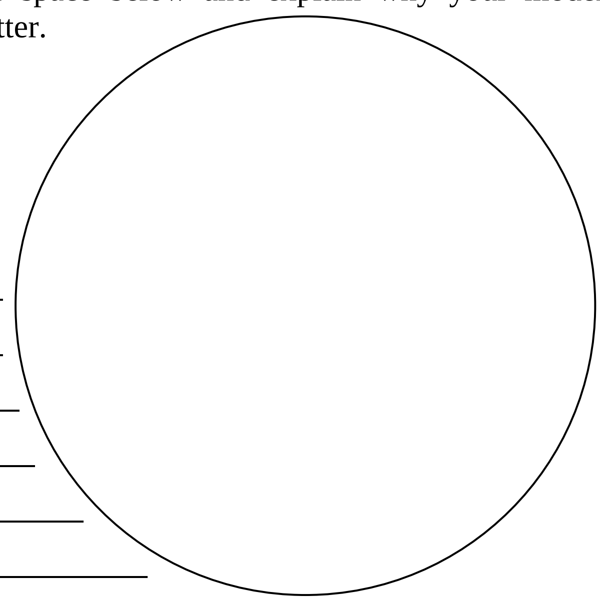


Model A



Model B

Create your hypothesis of the ideal model to represent the growth of a tree. Sketch the model in the space below and explain why your model is better.



2. What does ***Model A*** say about the growth of the tree?

3. What affects the growth of a tree? How do they affect the area of the tree circle? __________________

4. Using the formula for the area of a circle, find the ***instantaneous*** rate of change of the area A of a circle with respect to its radius r. Use the TI-89 graphing calculator and the formula:

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

(To use the TI-89, press $\boxed{\text{F3}}\boxed{3}$ to select limit(. Then enter (f(a+h)-f(a))/h, followed by $\boxed{,}$ *variable* $\boxed{,}$ $\boxed{)}$ $\boxed{\text{ENTER}}$.)

Write the answer here: $\dfrac{dA}{dr} =$

5. Verify the rate of change using the derivative command on the TI-89 graphing calculator. Is your answer from question 4 equivalent to the answer from question 5. (To use the TI-89 to find the derivative, press $\boxed{\text{2nd}}[d]$and enter the *function* followed by $\boxed{,}$ *variable* $\boxed{)}$ $\boxed{\text{ENTER}}$.

Write the answer here: $\dfrac{dA}{dr} =$

6. Keeping the change in area constant, which means that the area of the cross section of the tree increases by the same amount each year, use the formula for question 4 explain why the radius decreases as the area (of the rings) stays constant?

____

____

____

____

____

7. Comparing your prediction in question 1 to the information in question 6 (where all rings have equal areas), what changes would you make to your hypothesis?

____