NUMB3RS Activity: The Power of Powers
Episode: "Scorched"

Topic: Counting techniques   **Grade Level**: 7 – 12
Objective: Students will discover how vast amounts of knowledge can be gained from a
relatively small number of characteristics
Time: 10 – 15 minutes

Introduction

In "Scorched," Charlie explains that when fires are investigated, the investigators study
several elements that come together and form a signature for the fire. But unlike a
handwritten signature, the elements of a fire's signature are more general and less
individually specific. Elements like scorch marks and gasoline remnants are two different
characteristics of an arson-caused fire – the fire either exhibits the characteristic or it
does not.

Charlie characterizes fires according to 600 different characteristics. In this activity,
students will explore how the number of different characteristics considered causes the
number of possibilities to increase exponentially. This means that the number of
possibilities is much greater than the number of individual characteristics that are
examined.

Discuss with Students

A simple example can be used to demonstrate how fast the number of possibilities
changes for each characteristic. Suppose you ask a person whether they like ice cream
or not. There are two possible answers, "yes" or "no." What if you also ask whether or
not they like broccoli? What possible responses could you have? The person could say
"yes" to ice cream and "yes" to broccoli, or "yes" to ice cream but "no" to broccoli. With
two foods, you have four possible responses (YY), (YN), (NY), and (NN). For three
foods, there are eight possible responses and so forth.

If the answer to each question is not "yes" or "no" but rather "yes," "no," or "sometimes,"
you have just increased your possible responses significantly. For three foods, you
would have 27 possible responses!

Students should understand that the characteristics studied in this activity can be
ordered arbitrarily. However, once the characteristics are ordered, it is important that
they be kept in the same order to make sure that there are no duplicates. In our example
above, (YN) means that the person said "yes" to ice cream and "no" to broccoli. This
order is different than (NY) where the person said "no" to ice cream and "yes" to broccoli.

Depending on the level of the students, you can tie this activity into the study of
exponents and powers, logarithms, counting (combinatorics), probability, and other
number systems (like the binary number system).

Student Page Answers   **1**. *(000), (001), (010), (011), (100), (101), (110), (111)*   **2**. *8*
3. *number of possibilities = 2^n*   **4**. *There are 16 possibilities: (0000), (0001), (0010), (0011),
(0100), (0101), (0110), (0111), (1000), (1001), (1010), (1011), (1100), (1101), (1110), (1111)*
5. *The exact answer is 2^{600}, which will cause an overflow error when entered in most calculators.
To estimate 2^{600} you can use properties of exponents: $2^{600} = (2^{300})^2$. Then, using a calculator you
can estimate 2^{300} as 2×10^{90}. So, $2^{600} = (2^{300})^2 \approx (2 \times 10^{90})^2 = 4 \times 10^{180}$* **6**. *32*   **7**. *10*

Name: _____   Date: _____

NUMB3RS Activity: The Power of Powers

In "Scorched," Charlie explains that when fires are investigated, the researchers study several elements that come together and form a signature. Elements like scorch marks and gasoline remnants are two different characteristics of an arson-caused fire – the fire either exhibits the characteristic or it does not.

For example, consider whether or not there are scorch marks in an arson-caused fire. This would be considered one characteristic. The fire either has the characteristic (scorch marks) or it does not. If we represent the presence of a characteristic as 1 and lack of it as 0, the two possibilities are 0 and 1. If there are two characteristics (scorch marks, gasoline remnants), the fire could have neither characteristic (0, 0), the first and not the second (1, 0), the second and not the first (0, 1), or both (1, 1). So, two characteristics allow us to distinguish four different possibilities.

1. List all the possibilities for three fire characteristics. Let 1 represent the fire having the characteristic and let 0 represent the fire not having the characteristic.

2. How many possibilities did you find?

3. Look for a pattern in the three cases so far. Write a formula that tells how many possibilities there are for *n* characteristics.

4. Test your formula for *n* = 4 by listing all of the different possibilities.

5. **Challenge:** In "Scorched," Charlie investigates a fire with 600 characteristics. Use a calculator to estimate the number of possibilities in scientific notation. If you get an overflow error, use properties of exponents to find a different way to find an estimate.

Problems like the ones above can be generalized for different types of situations, where there are two choices for each characteristic. Use what you have learned in 1–4 to help you with 6 and 7. The total number of possibilities can be calculated if you know how many characteristics, given that each has two choices.

6. To indicate a small number, people sometimes say "you can count the number of times something happened on the fingers of one hand." In fact, if one "counts" by holding each finger in either the up or down position, how many different numbers can be indicated? (Hint: There are five characteristics, and for each, you have two choices: "up" or "down")

7. **Challenge:** Some years ago, a big burger chain advertised that they served their burgers 1,024 different ways. This was true because they offered a plain burger and a certain number of toppings which the customer could choose to put on or refuse. How many toppings were there to choose from?

The goal of this activity is to give your students a short and simple snapshot into a very extensive mathematical topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

Introduction
In this activity, you have found the number of possibilities when there are two choices for each characteristic, and you have examined how the number of possibilities changes based on the number of characteristics considered. A more general case of this type of problem is when there are more than two choices for each characteristic. Here, you will examine several of these types of cases, and how the number of possibilities is affected.

For the Student
- A simple way to think of a strand of DNA is as a chain of molecules. Each position of this chain can be one of four different chemical bases, which are abbreviated as A, C, T, and G. Use what you have learned in the activity to explore the number of ways these four bases could be arranged. How many ways are there to use these four bases to make a chain of 2 bases? of 3 bases? of 100 bases?

- Police often use a person's fingerprints as a form of identification. Although studying fingerprints is a complex subject, fingerprints are first classified into three basic categories – arch, whorl, and loop. Using these 3 possibilities for each fingerprint, how many different basic sets of fingerprints are possible for 1 hand? for both hands?

- For many characteristics, there are many more than two choices. For example, a person's hair color could be black, blonde, gray, etc. and a person's eyes could be blue, green, brown, etc. In general, how does the number of choices for a characteristic affect the formula you found in Question 3 on the student page? That is, what is the general formula if there are n characteristics and p possibilities for each characteristic?

- In many real-life situations, the number of possibilities for each characteristic varies. How does this affect the formula for the number of combinations?