TI-*nspire*™

Grade level: 9-12
Subject: Mathematics
Time required: 45 minutes

Shortest Distances

by Karen Droga Campe

Activity overview

In this activity, students will explore three situations involving distances between points and lines. First, the minimum distance between two points leads to the Triangle Inequality Theorem. Then, the shortest distance from a point to a line is investigated. Finally, students find the smallest total distance between two points on one side of a line and a point on the line.

Concepts

- *Minimum distance*
- *Triangle Inequality Theorem*
- *Perpendicular distance*
- *Reflections*

Teacher preparation

- *The Triangle Inequality Theorem states that the sum of the lengths of any two sides of a triangle must be greater than the length of the third side. Therefore, the distance between two points is shortest when it is a straight segment.*
- *The shortest distance from a point to a line is a segment from the point perpendicular to the line.*
- *The smallest total distance between two points on one side of a line and a point on the line occurs when the point on the line creates an angle of incidence that is equal to the angle of reflection. This occurs when the point on the line is collinear with one of the points off the line and the reflection of the other point over the line.*

Classroom management tips

- *This activity is designed to be* ***student-centered*** *with the teacher acting as a facilitator while students work cooperatively. Use the following pages as a framework as to how the activity will progress. Alternatively, the three problems can be explored as a teacher demonstration.*
- *The student worksheet provides a place for students to record their answers and observations.*
- *The Document Settings for the TI-Nspire can be accessed by pressing* (⌂on) (5) (2) (2) *(**Home > Settings & Status > Settings > Graphs & Geometry**). Geometry Angle should be set to "Degree" and the desired Display Digits can be set as well.*

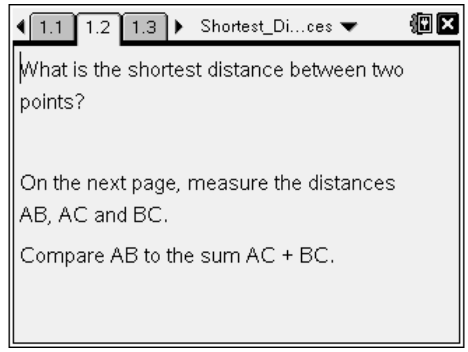
TI-Nspire Applications

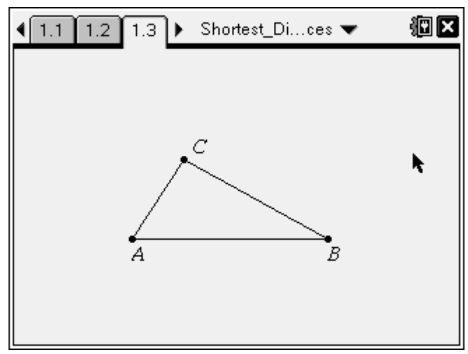
Graphs & Geometry, Notes

Problem 1 – Shortest Distance Between Two Points

Step 1: Have students open the file *Shortest Distance*. Read the directions on page 1.2.

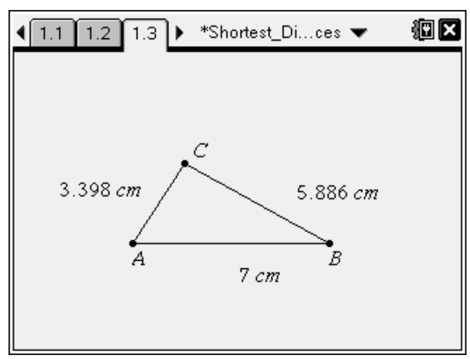
What is the shortest distance between points A and B?





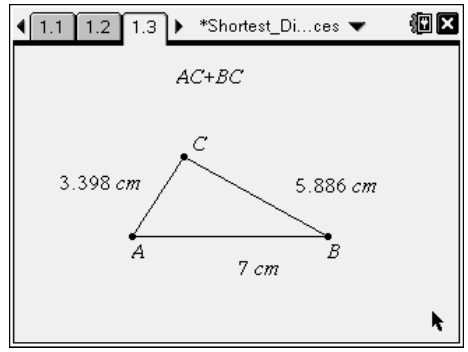
Step 2: On page 1.3, measure the distances AB, AC and BC.

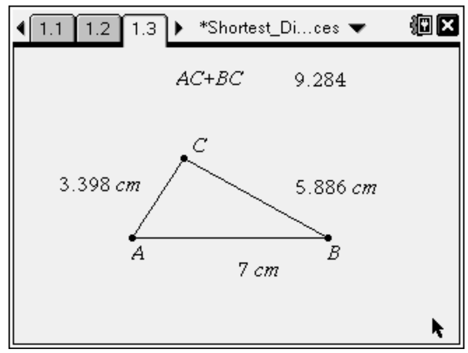
Drag point C around and observe the measurements.



Step 3: Use the **Text** tool to put the expression AC + BC on the screen.

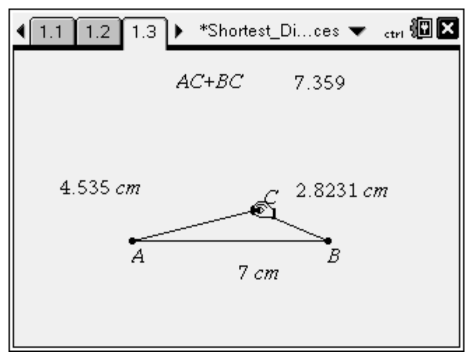
Use the **Calculate** tool to find the sum AC + BC.





Step 4: Drag point C around and compare the length of AB to the sum AC + BC.

Make a conjecture on the worksheet.



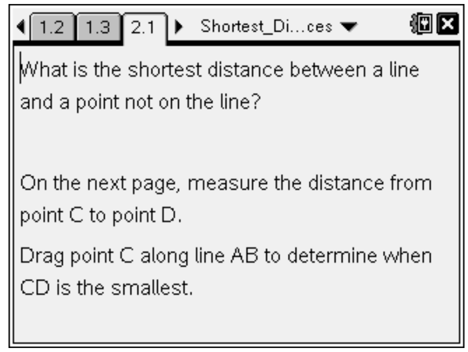
Step 5: Where does C need to be located for the three segments to form a triangle?

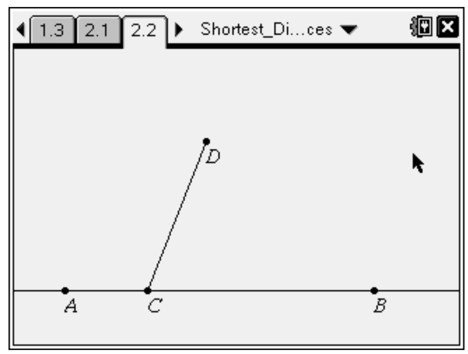
When there is no triangle formed, what is true about the lengths AB, AC, and BC?

Problem 2 – Shortest Distance From a Point to a Line

Step 1: Advance to page 2.1 and read the directions.

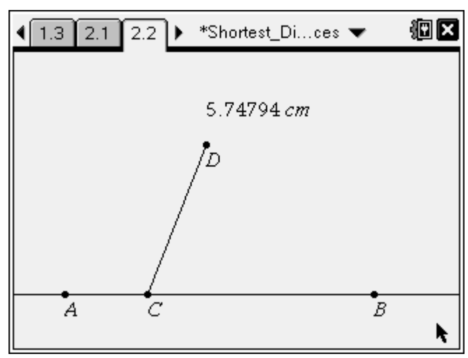
What is the shortest distance between point D and line $\overleftrightarrow{AB}$?



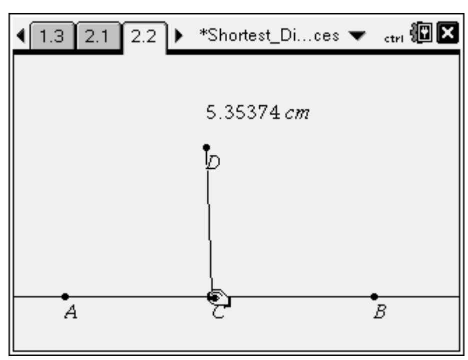


Step 2: On page 2.2, measure the distance from point C to point D.

Drag point C along line AB to determine when CD is the smallest.



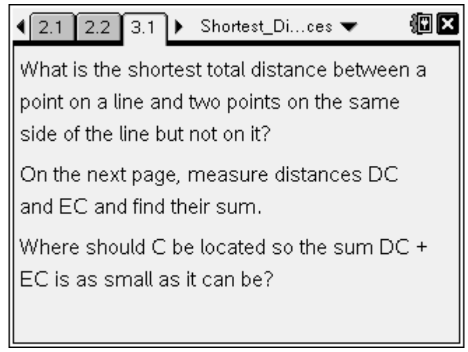
Step 3: What else is true about the figure when CD is the smallest? Make other measurements to support your conjecture.

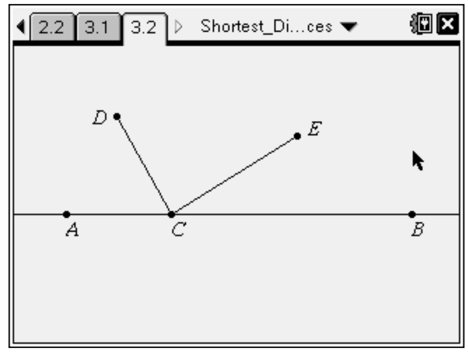


Problem 3 – Smallest Total Distance From Two Points to a Line

Step 1: Advance to page 3.1 and read the directions.

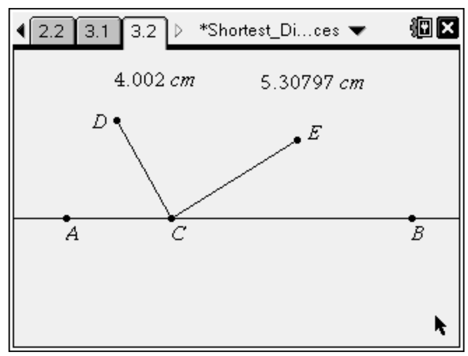
What is the smallest total distance between point C on the line and points D and E on the same side of line $\overleftrightarrow{AB}$?

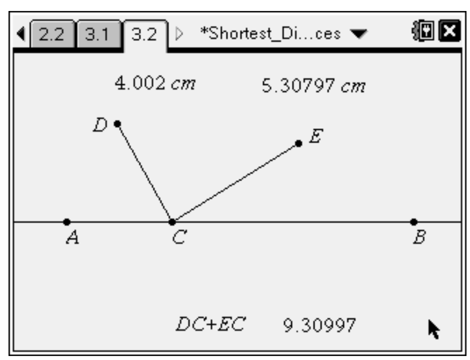




Step 2: On page 3.2, measure distances DC and EC and find their sum.

Drag point C so the sum DC + EC is as small as it can be.





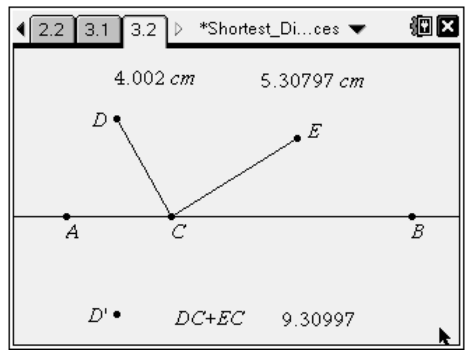
Step 3: Make some measurements of the figure.

Make a conjecture about what is true of the figure when the sum DC + EC is as small as possible.

Step 4: Change the locations of either points D or E or both, and find the minimum sum again.

Does this new figure support your conjecture?

Step 5: Reflect point D over line $\overleftrightarrow{AB}$. What does this new point have to do with the location of point C and the minimum sum?



Assessment and evaluation

- *In Problem 1, AB will always be smaller than the sum AC + BC, so long as C is off the line* $\overleftrightarrow{AB}$. *When C is off the line* $\overleftrightarrow{AB}$, *a triangle can be formed. When there is no triangle formed, C is on the line* $\overleftrightarrow{AB}$ *and AB = AC + BC (or one of the segments will equal the sum of the other two).*
- *In Problem 2, the shortest distance from point D to* $\overleftrightarrow{AB}$ *is when* $\overline{CD} \perp \overleftrightarrow{AB}$. *Students may measure either* $\angle ACD$ *or* $\angle BCD$.
- *In Problem 3, when the sum DC + EC is smallest, the angles between the segments and the line will be approximately equal. When point D is reflected across line* $\overleftrightarrow{AB}$, *the minimum total distance occurs when point C is collinear with point E and the reflection of point D. This relates to the Triangle Inequality Theorem as well: the distances DC and D'C are equal, so the shortest distance from D' to E is a straight line (the segments should not make a triangle).*

Activity extensions

- *Students can further explore the Triangle Inequality Theorem by creating 3 segments. Use the* ***Compass*** *tool to copy the segments and try to form a triangle. What must be true of the segment lengths in order to create a triangle? Can any three segments form a triangle?*
- *The result of Problem 3 has applications for the game of billiards. Given a cue ball and a target ball, where should the player aim in order for the cue ball to hit one bumper and then the target ball? What about two bumpers?*

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Materials: TI-Nspire

Student TI-Nspire Document

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