Exploring Quadratic Equations—Teacher Notes

Activity Overview

Students will stretch and translate the parabola given by $y = x^2$ and determine the effects on the equation. Students will also explore finding the vertex and zeros of a parabola and relate them to the equation.

Materials

- *Technology:* TI-Nspire handheld, TI-Nspire CAS handheld, or TI-Nspire computer software
- *Documents:* Explore_Quadratics.tns, Explore_Quadratics_Student.doc

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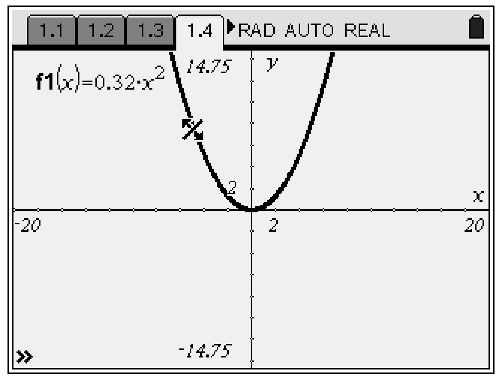
Problem 1 – Stretching a Parabola

In this problem, students are told $y = x^2$ is an equation in our library of functions and that the graph of $y = x^2$ is called a parabola. We can use the idea of transformations (shifting, stretching, and so on) to obtain other parabolas. Students will make a connection between the curvature of the parabola and the equation. Several questions follow to determine if students have made a connection.



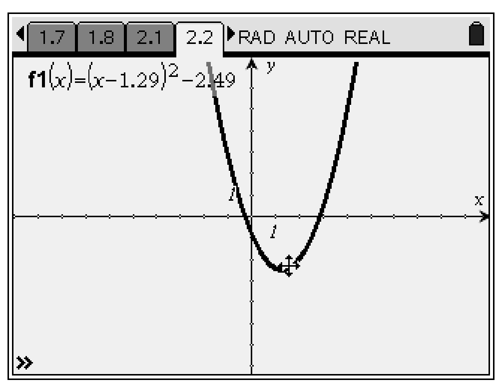
Note: To stretch the graph, the students will see the cursor change to ⤡. This means the graph can be grabbed and changed without changing the vertex.

Discussion Questions:

- Can you make the coefficient of x^2 negative?
- What happens to the graph if the coefficient of x^2 is between 0 and 1? Greater than 1? Less than 0?
- What happens if the coefficient of x^2 is 0?

Problem 2 – Translating a Parabola

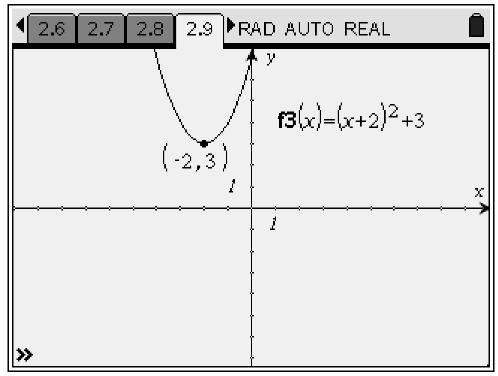
In this problem, students will translate the parabola $y = x^2$ by grabbing the vertex. Students will observe how the graph changes and make a connection between the vertex and equation. Several questions follow to determine if students have made a connection.



Discussion Questions:

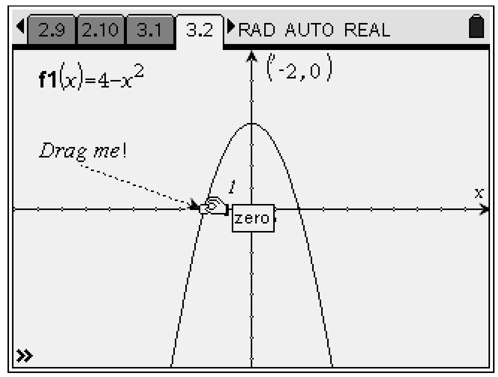
- How is the equation different when the vertex is in the first quadrant compared to when it is in the second quadrant?

- For a parabola written in the form $y = a(x - h)^2 + k$, what happens to the graph of $y = x^2$ when the constant k is positive? Negative?
- What happens to the graph of $y = x^2$ when the constant h is positive? Negative?
- How can we change the equation to standard form?



Problem 3 – Finding Zeros of a Quadratic Graphically

In this problem, the students will move a point on the graph of a parabola to find the zeros and the maximum/minimum. Students will answer a question about the zeros found in the exploration.

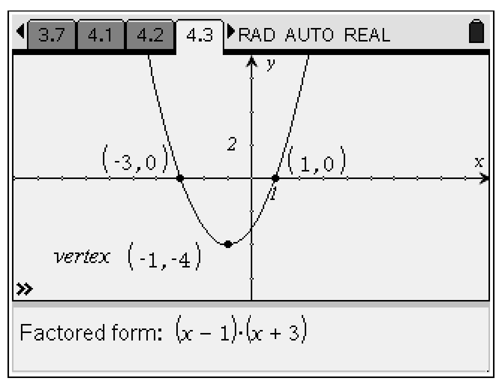


Discussion Questions:

- What is similar about the coordinates of the points representing the x-intercepts?
- How does the x-coordinate of the vertex relate to the two x-intercepts?
- What happens to the maximum/minimum when there is only one intercept?
- How can we algebraically find the zeros of the functions?

Problem 4 – Connecting Zeros to the Equation

In this problem, students will find the zeros of the parabola by finding the intersection of the parabola and the x-axis. Students will see the factored form of the quadratic equation and draw a connection between the zeros and the factored form. Students will then view the intercept form of a quadratic equation to determine how to use this form to find the zeros of the function without a graph.



Discussion Questions:

- How can we use the factored form of the quadratic equation to find the zeros?
- Is there an algebraic way to find the zeros?
- How can you find the zeros of a quadratic without the graph?
- How do we change the equation from intercept form to standard form?
- The x-coordinate of the vertex can be found by averaging the zeros.

Student Solutions

1. **a.** As coefficient of x^2 increases in magnitude the graph gets skinnier.

b. When coefficient is negative the graph opens down.

c. Negative since it opens down.

d. Any positive number will suffice.

2. **a.** Left and right shifts change the number inside the parentheses and up down shifts change the number on the end.

b. h is the *x*-coordinate of the vertex and *k* is the *y*-coordinate.

c. (–4, –2)

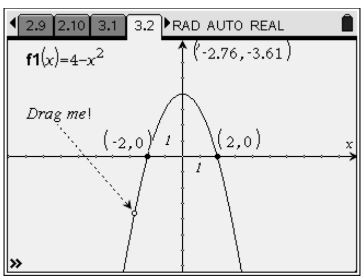
d. (3,1)

e. $c(x) = -3(x + 1)^2 + 1$

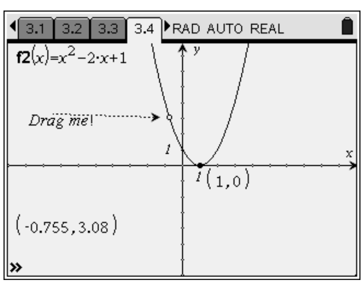
f. $f(x) = 2(x + 2)^2 + 3$ answers will vary

g. $f(x) = 5(x + 2)^2 + 3$ answers will vary

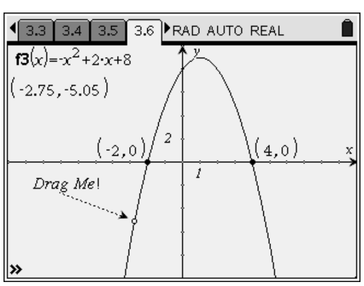
3. **a.**



b.



c.



4. **a.** *p* and *q* are the zeros.

b. –2 and 4. Must be of the form $a(x + 2)(x - 4)$

c. The *x*-coordinate of the vertex is the average of the zeros.