Grade level: Pre-service/secondary
Subject: Algebra
Time required: 105 to 120 minutes

TI-Nspire Activity: Number of Regions: An Introduction to the TI-Nspire CAS
By: Melissa Sutherland, State University of New York at Geneseo

## Activity Overview

This activity uses the problem of determining the maximum number of regions formed by $n$ lines in a plane to introduce new users to the TI-Nspire CAS. It illustrates the variety of features available on the TINspire CAS, as well as the ability to look at one problem from multiple representations (numerically, graphically, and analytically).

## Concepts

Pattern recognition, scatter plots, regression, hand fitting curves, solving systems of equations with matrices, multiple representations, reduced row echelon form, quadratic functions

## Teacher Preparation

I suggest explaining the Rule of Three to students - the idea of looking at a single problem from numerical (tables, numeric calculations), graphical/geometrical, and analytical (algebraical) perspectives. This is a key strength of the TI-Nspire and its ability to link these multiple perspectives across a problem. Since this is intended as an introduction to the TI-Nspire, it would be helpful to give a brief overview of the document structure of the handheld and the types of application pages possible. It would also be helpful to discuss the keypad and the variety of levels that the keys are used at. Students who are familiar with some dynamic geometry program should have an easier time with that page, although directions are given for inserting lines and manipulating them. Be sure to stress that students should read all the directions. While the .tns file is written to be self contained, it may be too difficult for your students to read the directions on the note pages and have to scroll and move to another page to implement the directions. Therefore, the Student Worksheet contains all the directions contained in the .tns file in a way that will be easier to read page by page.
-This activity uses all applications: Notes, Graphs \& Geometry, Lists \& Spreadsheets, Data \& Statistics, and Calculator.
-Getting started tasks
Open the document.
Perhaps give a quick explanation of the problem.
Remind students that the file is self contained but if they prefer to not scroll to read the directions, they can find them on the student worksheet.
Remind students that they will be recording answers along the way on the student worksheet. Students might need help remembering that they should use ctrl and the NavPad to switch from page to page.
Remind students that they need to scroll down on any Notes page until they see "end of page".
Remind students to read all directions.
Below is a page by page description of the activity and screen shots of each page in the activity.

On Page 1.1 students are introduced to the problem of putting lines in a plane and counting the maximum number of regions that it is possible to produce. Consider discussing the problem with them to be sure they are clear what we will be counting.

On Page1.2, students are told about the Graphs \& Geometry application on Page 1.3 and that it is in Plane Geometry view. Students are told how to use the commands found under menu and Points and Lines on page 1.3 to graph 1 line, 2 lines, 3 lines and 4 lines in the plane, each time counting the maximum number of regions they can make. Students should be told to make their lines long enough to fill the screen to make determining the number of regions easier. They also should be told how to grab and drag a line if they need to produce more regions. Directions for doing this are included in the activity and also on the student worksheet. Be sure to emphasize adding one line at a time. Students are told to record the maximum number of regions for $0,1,2,3$, and 4 lines on the table on page 1.4.

Page 1.3 is a Graphs \& Geometry page in Plane Geometry view. Students investigate the maximum number of regions that are determined by placing 1 line, 2 lines, 3 lines, and 4 lines in this plane. It would probably be helpful if students had some experience playing on such a page and drawing lines and manipulating them.

Page 1.4 is where students record the data they collected about the maximum number of regions they are able to produce for $0,1,2,3$, and 4 lines. It would be good to check that students have the correct data before telling them to continue in the activity.


Page 1.3 is a Graphs \& Geometry page in Plane Geometry View. If you have used Geometer's Sketchpad or Cabri before, you should find familiar tools under the menu key. On page 1.3, you will add lines one at a time (see Hints below for how to do this), moving them around to create the maximum number of regions.

As you do, count the maximum number of


On Page 1.5, we discuss the desire to find a formula to compute the maximum number of regions determine by any number of lines without actually having to construct the lines and count them. Students are asked to look back at the table on Page 1.4 and determine if the relationship is linear and to explain their response on the student worksheet.

Page 1.6 explains that the data entered on page 1.4 will now be in a table on page 1.7 in a split screen with a Data \& Statistics application. It gives directions for making a scatter plot of the data in the table and explains how to toggle between areas in a split screen.

Page 1.7 is a split screen with the Lists \& Spreadsheet application on the left which will automatically contain the data the students entered on page 1.4 and a Data \& Statistics application on the right where they will create a scatter plot of their data points.

Page 1.8 contains directions for learning how to alternatively graph the data points in a Graphs \& Geometry page by changing the Graph Type to Scatter Plot. The goal is to show the students two different applications in which they can make scatter plots.


| 1.4 | 1.5 | 1.6 | 1.7 | RAD AUTO REAL |
| :--- | :--- | :--- | :--- | :--- | Let's graph our data points to determine what type of function we have.

Page 1.7 has a split screen containing a
Lists \& Spreadsheet application and a Data
\& Statistics application. You will find that all
the data you entered in the table on page 1.4 is now also on page 1.7. To toggle between the parts of the split screen, use the ctrl key and the tab key to move from the Lists \&


| 1.5 | 1.6 | 1.7 | 1.8 | RAD AUTO REAL |
| :--- | :--- | :--- | :--- | :--- |

You can also plot the data points in a Graphs
\& Geometry application as in page 1.9. To try this, on page 1.9 , under the menu key, select Graph Type and Scatter Plot. At the bottom, use tab to go from one variable to the other, hitting enter and the NavPad to select select the variable lines to store in $x$ and the variable regions to store in $y$.
end of page

Page 1.9 is a Graphs \& Geometry page where students plot their data points in a scatter plot.


##  What type of function does it appear might best fit our data points (at least for positive integer values of $x)$ ? <br> Doesn't it seem to be quadratic? <br> To investigate this, we'll graph $f(x)=x^{2}$ on the same axes as our scatter plot on page 1.9 and drag the graph to hand fit it to our data points. To do this, on page 1.9 , under the menu key, select Graph Type and

| 1.8 | 1.9 | 1.10 | 1.11 |  |
| :--- | :--- | :--- | :--- | :--- | :--- |

Now we'll see how to do a quadratic regression using our data points and then we'll compare the regression to our hand fit quadratic. We'll do this in two different applications so you can see how to do regression in both.

First, we'll use the Lists \& Spreadsheet
application. You'll find a copy of your table


Page 1.12 is a Lists \& Spreadsheet application which will automatically contain the data points entered on page 1.4. Students will do a quadratic regression and the output go into columns C and D in the spreadsheet. You might need to help the students read off the coefficients for the quadratic.

Page 1.13 gives directions for adding a page to our document, in particular, a Data \& Statistics page. This added page will then be page 1.14. On page 1.13, students are given directions for how to do quadratic regression on this new Data \& Statistics page versus a Spreadsheet. Again, the goal is to give students options for dealing with their data in the future.

Page 1.14 (It will become page 1.15 after students insert the Data \& Statistics page as directed on page 1.13.) is a split screen with directions on the left for how to use the Calculator application on the right to expand the hand fit quadratic so it can be easily compared to the quadratics found through regression. Students learn how to use the catalog to find the command "expand" under the Algebra commands. This requires the CAS. The quadratic found from regression is stored in f2. Students will expand $f 1(x)$ and then look again at $\mathrm{f} 2(\mathrm{x})$ to see how well they did in hand-fitting a quadratic to our data points.

Page 1.15 (will be 1.16 after inserting a new 1.14 page) directs students to take three of their data points and write a system of three quadratic equations in three unknowns to find the quadratic $A x^{\wedge} 2+B x+C=y$ passing through those three data points using algebra where $x$ is the number of lines and $y$ is the maximum number of regions formed. Students are asked to record their system of equations on the student handout. Students are also given directions for putting the coefficients of these equations into the matrix on page 1.16 (will be page 1.17) so that the matrix can be put into reduced row echelon form. Students are asked to record the A, B, C found in the reduced row echelon form of the augmented matrix representing our system of equations.

Page 1.16 (will be 1.17) is a Calculator page which is set up for students to input coefficients into an augmented matrix representing their system of three equations with three unknowns and find its reduced row echelon form.
 Next we'll show how to add a page to a problem, in this case a Data \& Statistics page, so we can explore an alternate way to get a quadratic regression of our data points. To do this, hit the ctrl key and then the Home key, to get the Tool menu. Select Insert and
Page. Choose Data \& Statistics.
Click on the axes to select lines for the $x$ -axis and regions for the $y$-axis.


Let's now use algebra to find the quadratic funtion passing through our data points.
Let $x$ be the number of lines in the plane.
Let $y$ be the maximum number of regions determined by the $x$ lines.

We want to find coefficients $A, B$, and $C$ such that $A x^{2}+B x+C=y$.


## Assessment and Evaluation

Make sure students have recorded their answers on the student worksheet. My goal was just an introduction to the TI-Nspire CAS and not assigning a grade to the activity.

See Student Worksheet and screen shots for possible responses for the Student Worksheet Solutions.

