

Name _____

Date _____

EXPLORATIONS

ACTIVITY 8

Figuring Areas

A polygon can be defined as a closed figure with n number of sides. A polygon is *regular* if all its sides are congruent and all its interior angles are congruent. No matter how many sides a regular polygon has, its interior area can be divided into the most basic polygon—triangles. When these triangles are formed with a common vertex at the center of the polygon, they are both congruent and isosceles. Finding the area of a regular polygon, then, can be simplified to finding the sum of the areas of these triangular regions.

In this activity, you will figure the areas of regular polygons using a summative approach and then compare your summations with the areas calculated by the Cabri Geometry Application. Patterns of area and perimeter values will be examined as the number of sides of the polygon is changed.

Instructions

1. Press **[ON]** to turn on the Voyage™ 200 PLT. To reset to the default settings, press **[2nd] [MEM] [F1]**, select 1:RAM, select 2:Default, and press **[ENTER] [ENTER]**.

2. To begin a geometry session, press **[APPS]**, select Cabri Geometry, and then select 3:New.

In the NEW dialog box, press **⌵** to move and type **AREA** for the variable name. Press **[ENTER] [ENTER]** to save.

3. Construct a regular pentagon using a radius specified by the length of a segment as follows:

Press **[F2]**, and select 5:Segment. Move the cursor to the center of the screen and press **[ENTER]**, and then press **A** to label this endpoint of the segment as **a**.

Move the cursor to the right to lengthen the segment to about 1.5 cm (around $\frac{3}{4}$ inch) and press **[ENTER]**. Press **B** to label this point **b**.

***Note:** The label **a** needs to be repositioned so that it doesn't interfere with later parts of the construction. Press **[ESC]** and move the cursor to the letter **a**. When the message "**THIS LABEL**" appears, hold down **[GRAB]** (grab) and use **⌵** to place the label above the point and then release.*

Figuring Areas (Continued)

- To draw a regular pentagon, press **[F3]** and select 5:Regular Polygon. Move the cursor to endpoint a of the segment. When the message “THIS POINT” appears, press **[ENTER]** to fix the center point of the polygon.

Move the cursor to the right to endpoint b. When the message “THIS POINT” appears, press **[ENTER]** to fix the radius of the polygon.

A dotted circle with a chord appears, and the number of sides of the regular polygon is displayed at the center point in braces (the default value is six). To decrease the number of sides to five, move the cursor clockwise from its current position until {5} appears. Then press **[ENTER]** to complete the polygon.

- Starting with radius ab already drawn, construct a triangle with two additional segments as follows:

Press **[F2]** and select 5:Segment. Move the cursor to point b, and press **[ENTER]** when the message “THIS POINT” appears. Draw the segment to an adjacent vertex on the polygon, and press **[ENTER]** after receiving the message “THIS POINT.”

Use the same procedure to draw the third segment of the triangle from this point back to center a.

To label the third vertex of the triangle as c, press **[F7]**, and select 4:Label. Position the cursor at the appropriate point on the polygon so that the message “THIS POINT” appears and the cursor changes from a plus to a vertical bar. Then press **[ENTER]** and type **C**. Press **[ESC]** to exit the command.

- To construct an altitude of the triangle, which is also called the *apothem* of the regular polygon, press **[F4]** and select 3:Midpoint.

Move the cursor to the base of the triangle, bc, and press **[ENTER]** when you see the message “WHICH OBJECT?” Select “2:THIS SEGMENT” to place a midpoint on the segment.

Press **[F2]** and select 5:Segment. Connect the midpoint you just made to point a of the circle to complete the triangle’s altitude (polygon apothem).

Figuring Areas (Continued)

7. Now, measure the length of the base and altitude of the triangle, as well as radius ab of the polygon.

Press $\boxed{F6}$ and select 1:Distance & Length. Move the cursor to base bc of triangle abc . When you see the message "WHICH OBJECT?" press \boxed{ENTER} and select "2:THIS SEGMENT" to find the length in centimeters. Then measure the lengths of the altitude and radius in this order.

Record the length of each segment below.

<i>base</i>	<i>altitude</i>	<i>radius</i>
<i>length =</i> _____	<i>length =</i> _____	<i>length =</i> _____

8. The area of triangle abc can be determined by traditional means using the formula: $A = \frac{1}{2}BH$, where B represents the base and H represents the height of the triangle.

To do this calculation on the screen, press $\boxed{F6}$ and select 6:Calculate. A calculation entry line appears at the bottom of the screen.

In the entry line, press $\boxed{[] 1 \div 2 []}$. Press \odot to highlight the length of base bc , and press \boxed{ENTER} to copy this value, a , to the entry line.

Press $\boxed{\times} \odot \odot$ to highlight the length of the altitude. Press \boxed{ENTER} to copy this value b , and then press \boxed{ENTER} to find the answer to the calculation. The result, R , is displayed in the bottom left corner of the screen.

Record the area of the triangle below.

area of triangle abc = _____

9. The interior area of the regular pentagon can be further divided into triangles congruent to the one constructed above. What is the total number of triangular areas that can be formed in the pentagon? Write this number below.

number of triangles = _____

Figuring Areas (Continued)

10. To determine the area of the entire polygon, you can multiply the area of the triangle, $A = \frac{1}{2}BH$, by the number of triangles, n , that can be formed in the polygon.

$$\text{Area of a regular polygon} = (\frac{1}{2}BH)(n).$$

To perform this calculation, press $\odot \odot \odot \odot$ to highlight the result value corresponding to the area of the triangle found in step 8. Press $\boxed{\text{ENTER}}$ to copy this value, a , to the entry line.

Press $\boxed{\times}$ and type the number recorded in step 9. Press $\boxed{\text{ENTER}}$ to find the area of the pentagon, and record this value below.

area of pentagon =

11. The mathematical formula for calculating the area of any regular polygon is given by:

$$\text{Area of a regular polygon} = \frac{1}{2}aP,$$

where a stands for the apothem (the perpendicular segment from the center of the polygon to a side—altitude of a triangle) and P stands for perimeter.

Explain why $\frac{1}{2}aP$ is equivalent to $(\frac{1}{2}BH)(n)$.

To find the perimeter of the pentagon, press $\boxed{\text{F6}}$ and select 1:Distance & Length. Move the cursor to a side of the regular polygon (other than side bc), and press $\boxed{\text{ENTER}}$ when the message “PERIMETER OF THIS POLYGON” appears.

Record this value below.

perimeter of pentagon =

Figuring Areas (Continued)

12. On the Voyage™ 200 PLT, calculate the area of the pentagon using the new formula in step 11. Press $\boxed{F6}$ and select 6:Calculate.

In the entry line, press $\boxed{[] 1 \boxed{=} 2 \boxed{]}$. Press $\leftarrow \leftarrow$ to highlight the altitude length of the triangle and then press \boxed{ENTER} to copy this value, a, to the entry line.

Press $\boxed{\times} \leftarrow \leftarrow \leftarrow \leftarrow \leftarrow \leftarrow \leftarrow$ to highlight the perimeter of the polygon, and press \boxed{ENTER} to paste this value, b, to the entry line.

Press \boxed{ENTER} to find the area of the pentagon. Record this value below.

area of pentagon =

How does this area value compare with the one found in step 10?

13. The Voyage 200 PLT can automatically find the area of a polygon. Press $\boxed{F6}$ and select 2:Area.

Move the cursor to a side of the pentagon (other than bc), and press \boxed{ENTER} when the message "THIS REGULAR POLYGON" appears. Record this area below.

area of pentagon =

How does this value compare with the areas calculated in step 10 and step 12?

Figuring Areas *(Continued)*

14. Now that you have investigated three methods for finding the area of a regular polygon, let's explore how these areas and perimeters change as the number of sides increases. Follow the steps below to begin a new construction for each of the polygons listed in the table below.

- (a) Press **[ESC]** and move the cursor to the first vertex clockwise from b. When the message "THIS POINT" appears, press **[ENTER]** and then **[←]**. The entire figure is erased except for radius ab. The new polygon will be constructed with this segment to ensure that it has the same radius measure as the original pentagon.
- (b) Follow the procedure in step 4 to construct the regular polygon.
- (c) Find the perimeter and area of the polygon as outlined in step 11 and step 13. Record these values in their appropriate spaces in the table.
- (d) Repeat (a) through (c) for each polygon in the table below. You do not need to repeat the construction for the pentagon; simply use the values recorded in the activity.

regular polygon	perimeter (cm)	area (cm ²)
triangle		
pentagon		
octagon		
dodecagon		
17-gon		

15. As the number of sides of the regular polygon increases, what geometric shape does the figure approach?

Figuring Areas (Continued)

16. The circumference (perimeter) of a circle is denoted by the formula, $C = 2\pi r$, where r represents the radius of the circle. Using the **Calculate** tool as follows, find the circumference of a circle with the same radius as the polygons.

Press $\boxed{F6}$ and select 6:Calculate. In the entry line, press $2 \times \boxed{2nd} [\pi]$. Press \odot to highlight the radius length, ab , and then press \boxed{ENTER} to copy this value, a , to the entry line.

Press \boxed{ENTER} to perform the calculation. Record the displayed result below.

circumference of the circle =

As the number of sides of the regular polygons increases, how do the perimeters compare with the circumference of the circle?

17. Now compute the area of the circle, given by the formula $A = \pi r^2$.

In the calculation entry line, enter $\boxed{2nd} [\pi] \times$. Press \odot to highlight the radius length, ab , and then press \boxed{ENTER} to copy this value to the entry line.

Press $\boxed{\wedge} 2 \boxed{ENTER}$ to compute the area. Record this result below.

area of the circle =

As the number of sides of the regular polygons increases, how do the areas compare with the area of the circle?

Figuring Areas (Continued)

Questions

1. If the triangular area decreases as the number of polygon sides increases, why does the total polygon area increase?

2. Explain why the formula $A = \frac{1}{2}aP$ cannot be used to find the area of a nonregular polygon.

3. Why was it important to construct each of the regular polygons in the table using the same radius measure?

4. Compare the perimeter and area values for the 17-gon and circle. Why is the difference between the area values greater than the difference between the perimeter values?

Hint: What mathematical operation is used to arrive at the area value for the circle that is not used for the circumference computation?