

## NUMB3RS Activity: Waxing Elliptical Episode: "Harvest"

**Topic:** Ellipses

**Grade Level:** 8 - 11

**Objective:** Students will investigate how the distance between foci of an ellipse and the shape of the ellipse are related.

**Time:** about 35 minutes

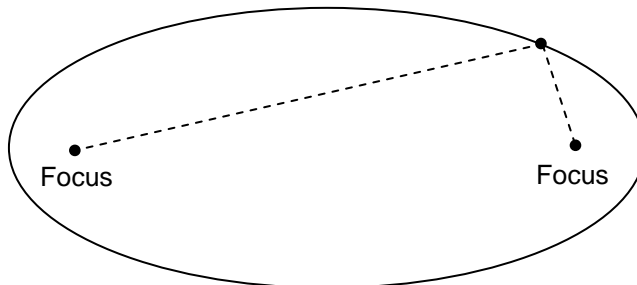
**Materials:** wax paper, large coffee can lid (or other traceable circle or a compass), ruler, fine-tip permanent markers, scissors, overhead projector

### Introduction

In "Harvest," Charlie is helping Don investigate illegal kidney transplants in Los Angeles. After analyzing the trip log of the ambulance suspected of transporting the kidneys, Charlie is able to determine that the ambulance went out of its way to go to an unknown location, traveling 26 miles when the straight distance was 20 miles. Assuming the ambulance traveled in straight lines, the unknown location must lie on an ellipse with the start and end points as the foci. Using this information, Charlie and Larry find the likely locations where the transplants occurred. In this activity, students will learn how the foci of an ellipse are related to the shape of the ellipse.

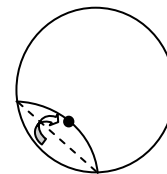
### Discuss with Students

An ellipse is the set of all points in a plane, the sum of whose distances from two fixed points (**foci**) is constant. For example, for any point on the ellipse below, the sum of the lengths of the dashed segments remains the same.



In this activity, students will generate ellipses by folding wax paper. Students will draw a large circle on a piece of wax paper and plot a point somewhere *inside* of the circle. Be sure the students' points vary from close to the circle's center to close to its edge.

Next, students will fold their circle so that it touches the plotted point and carefully crease their wax paper as shown in the figure. Students will then unfold the wax paper, choose another part of the circle and repeat the process. As they repeat the folding process using many other points around the edge of the circle, an ellipse will appear through the creases. Emphasize the importance of doing this process carefully.



**Student Page Answers:** 1. the center 2. As the points get closer to each other, the ellipse looks more like a circle. 3. The shape would be a circle. 4. These sums are all approximately the same. 5. If the foci were on top of each other, the figure would be a circle. The foci would be at the center of the circle, so the distance to the edge is the radius.

Name: \_\_\_\_\_ Date: \_\_\_\_\_

## NUMB3RS Activity: Waxing Elliptical

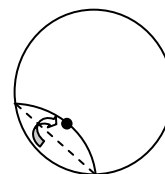
In "Harvest," Charlie is helping Don investigate illegal kidney transplants in Los Angeles. After analyzing the trip log of the ambulance suspected of transporting the kidneys, Charlie is able to determine that the illegal transplants occur at an unknown location somewhere on an ellipse. We know the positions of the foci and the distance from one focus to the other by way of the unknown location.

In this activity, you will use paper folding to create an ellipse, and locate its foci.

**Step 1:** Using a permanent marker, trace or construct a large circle on a piece of wax paper. Carefully cut out this circle.

**Step 2:** Place a point somewhere inside of the circle. Make sure your point is in a different location than other students' points.

**Step 3:** Carefully fold the wax paper so that a point on the edge of the circle touches the point you placed inside the circle. Fold the wax paper to form a crease (as shown at the right).



**Step 4:** Unfold the wax paper, and create a new fold (crease) so that a *different* point on the edge of the circle touches the point inside the circle. Repeat this process using many different points around the edge of the circle. After all the folding is completed, draw the outline of the ellipse on your wax paper.

Now use your figure to answer the following questions.

1. Fold your circle in half. Unfold, and repeat in a different direction. When it is unfolded, notice that your two folds are diameters of the circle. Mark the point of intersection of the diameters with the permanent marker.

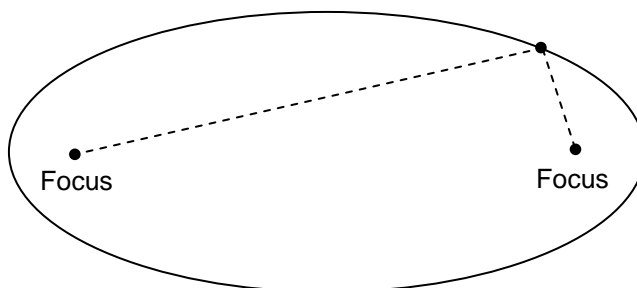
What is this intersection called? \_\_\_\_\_

2. Compare your ellipse to ellipses constructed by other students. What do you notice about the ellipse's shape and the distance between the two points?

\_\_\_\_\_

3. What do you think your figure would look like if the two points were on top of each other?
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Each of these two points is a **focus** of the ellipse, and the two **foci** determine the ellipse. An ellipse is defined as the set of all points in a plane, the sum of whose distances from the foci is constant. For example, for any point on the ellipse below, the sum of the lengths of the dashed segments remains the same.



Test this with your ellipse. Mark a point on your ellipse. Use your ruler to measure the distance from each focus to this point. Add these distances and record your result. Repeat this for three other points on the ellipse.

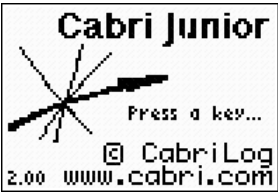
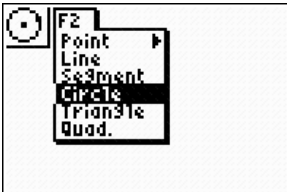
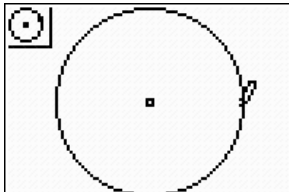
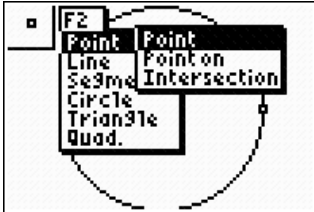
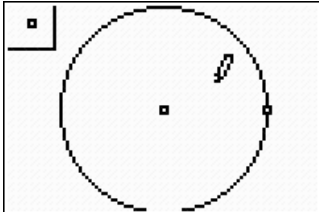
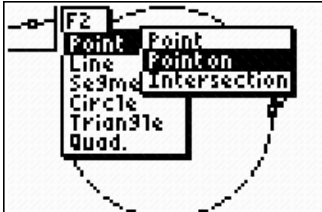
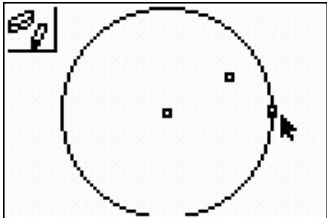
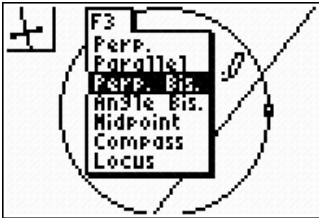
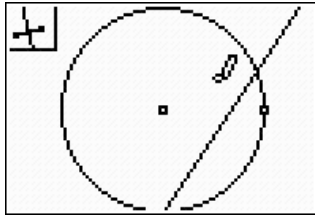
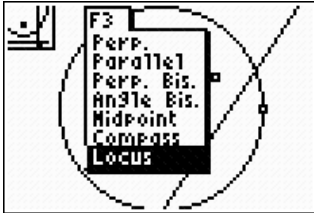
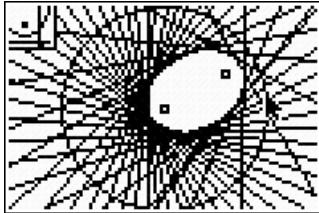
4. What do you notice about these sums? \_\_\_\_\_

5. Suppose the two foci were on top of each other. Describe how the distance from the foci to a point on the edge of the figure is related to the figure.
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*The goal of this activity is to give your students a short and simple snapshot into a very extensive mathematical topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.*

### Extension: Using Cabri® Jr.

The example below shows how to construct an ellipse using the Cabri Jr. App on a TI-84 Plus graphing calculator. (This App can be downloaded for free from <http://education.ti.com/cabrijr>.)

- 1  Press **[APPS]**  
Select CabriJr.
- 2  To make a circle, first press **[WINDOW]** and choose **Circle**.
- 3  Press **[ENTER]** to set the center of the circle and use the arrow keys to set the radius. Press **[ENTER]**.
- 4  Press **[WINDOW]** **[▶]** and choose **Point**. Press **[ENTER]**.
- 5  Use the arrow keys and press **[ENTER]** to set the point's location.
- 6  Press **[WINDOW]** **[▶]** **[▼]** to choose **Point on**. Press **[ENTER]**.
- 7  Use the arrow keys to locate a point somewhere on the circle. Press **[ENTER]** to plot the point.
- 8  Press **[ZOOM]** and choose **Perp. Bis.** Press **[ENTER]**.
- 9  Move to the point on the circle and press **[ENTER]**. Then move to the point inside the circle and press **[ENTER]**.
- 10  Press **[ZOOM]** and choose **Locus**.
- 11  Select the perpendicular bisector and then the point on the circle.

**For the Student**

In this activity, you used folding techniques to construct an ellipse. You can use similar folding techniques to create other figures.

- Repeat the activity, but this time place the point outside of the circle (do not cut out the circle). What figure is traced by the folds? How would the figure change if the point were located closer to the circle? Farther from the circle?
- Use the same procedure as in the activity, but this time draw a line and a point not on the line (do not draw a circle). Carefully fold the wax paper so that a point on the line touches the plotted point, and make a crease. Unfold the wax paper and create a new fold (crease) so that a *different* point of the line touches the plotted point. Repeat the process using many points on the line. What figure is traced by these folds? How would the figure change if the point were located closer to the line? Farther from the line?

**Additional Resources**

Ellipses are a part of a family of graphs called **conic sections**. You can learn more about ellipses and other conic sections at this Web site:

**<http://britton.disted.camosun.bc.ca/jbconics.htm>**

Another activity involving finding the foci of an ellipse can be found at:

**<http://johnbanks.maths.latrobe.edu.au/Games/Ellipse>**

A full explanation as to why an ellipse is formed can be found at

**[http://www.mste.uiuc.edu/courses/ci303fa01/students/elord/math\\_assignment1/conics.html](http://www.mste.uiuc.edu/courses/ci303fa01/students/elord/math_assignment1/conics.html)**