## About the Lesson

In this activity, students will explore multiplication and division of fractions using several representations to develop fluency.
Students will also solve a problem involving similarity using proportions. As a result, students will:

- Multiply and divide fractions.
- Multiply and divide decimals.
- Apply proportionality to solve similarity problems.
- Apply and extend previous understandings of multiplication and division to divide fractions by fractions.
- Solve real-world and mathematical problems involving four operations with rational numbers.
- Model with mathematics, use appropriate tools strategically, and construct viable arguments.


## Vocabulary

- similarity
- proportions


## Teacher Preparation and Notes

- Fraction tiles and graph paper will be useful in this activity.



## Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus C Silver Edition. It is also appropriate for use with the TI84 Plus family with the latest TI84 Plus operating system (2.55MP) featuring MathPrint ${ }^{\text {TM }}$ functionality. Slight variations to these directions given within may be required if using other calculator models.
- Access free tutorials at http://education.ti.com/ calculators/pd/US/Online-


## Learning/Tutorials

- Any required calculator files can be distributed to students via handheld-to-handheld transfer.


## Compatible Devices:

- TI-84 Plus Family
- TI-84 Plus C Silver Edition


## Associated Materials:

- Visualizing_Fractions_ Student.pdf
- Visualizing_Fractions_ Student.doc


## Part 1 - Multiplying Fractions

Students should be familiar with area models for multiplication. If they have not seen area models used for fractions, this might need a little extra time.

1. Multiply $\frac{4}{5} \times \frac{3}{5}$.

Press the green ALPHA key, then $Y$ for [F1] Choose the fraction template by pressing ENTER. Type the numerator 4.
Press 5 to type the denominator 5. Press to move out of the denominator and multiply. Press $\boxtimes$. Press ALPHA [F1] ENTER 3 to type the numerator of the second fraction, 3. Press 5 to type the denominator 5. Press ENTER.
a. Write the fraction. Answer: $\frac{12}{25}$
b. What percent of the 1 by 1 square is shaded?

Answer: 48\%
Students can find this several ways without a calculator.
They could set up the proportion $\frac{12}{25}=\frac{\square}{100}$

and find $\frac{12}{25}=\frac{12 \times 4}{25 \times 4}=\frac{48}{100}=48 \%$. Alternatively,
students can mentally compute $0.8 \times 0.6=0.48=48 \%$

With a calculator, you can use the Fraction to
Decimal conversion tool. Press ALPHA $Y$ for [F1] and choose the fourth option. Then multiply the decimal 0.48 by 100 .
2. Multiply $\frac{2}{3} \times \frac{1}{4}$.
a. Write the fraction. Answer: $\frac{1}{6}$

The 1 by 1 square consists of 12 fractional pieces, 2 of which are shaded. Ask students why the denominator is 6 instead of 12 (i.e. $3 \times 4$ ). They should explain that the fraction was reduced.

When using the calculator, students must enter the numerator, the fraction key, the denominator, the multiplication operation, and then the second fraction.
 ALPHA [F1] ENTER 1 - 4 ENTER.

If students forget to press they will continue entering in the denominator of the first fraction.
b. What percent of the 1 by 1 square is shaded?

Answer: 16.7\% (rounded to the nearest tenth.)
Key presses are ALPHA [F1] 4 EENTER $\otimes 1000$ ENTER.

3. Multiply $1 \frac{1}{2} \times \frac{3}{4}$.

To enter a mixed number, students will use the mixed number template. The keystrokes for this example are $\triangle \mathrm{ALPHA}[\mathrm{F} 1]$ and choose the second option. Then press 1 - 1 - 2 — ALPHA [F1] ENTER 3 - 4 ENTER.

To convert from an improper fraction back to a mixed number press ALPHA [F1] [3] ENTER.

After students work with using the area model to multiply fractions, put them in small groups. Have the groups discuss how the area model helps with multiplication.

The 1 by 1 square consists of 8 fractional pieces. Since the total shaded area consists of 9 fractional pieces, more than $100 \%$ of the 1 by 1 square is shaded. Students may notice that the total shaded area includes the whole 1 by 1 square ( 8 pieces) plus one additional fractional piece, $\left(\frac{1}{8}\right)$, so we have $1 \frac{1}{8}$.

Ask students why the numerator of the mixed number is 1 instead of 3 . A common mistake is to multiply the numerators of the two fractions without converting the mixed number to an improper fraction first.
a. Write the fraction. Answer: $1 \frac{1}{8}$
b. What percent of the 1 by 1 square is shaded?

Answer: 112.5\%
If using a calculator, students press ALPHA [F1] 4 [ENTER and multiply by 100. A common mistake is to report $1.125 \%$. Discuss how to find the percent without a calculator. The amount of shaded area is $100 \%$ (or 1 ) plus $\frac{1}{8}$. To write $\frac{1}{8}$ as a percent, students could set up the proportion $\frac{1}{8}=\frac{\square}{100}$ and utilize $100=4 \times 25$ :

$$
\frac{1}{8}=\frac{1}{2 \times 4}=\frac{1 \times 25 / 2}{2 \times(4 \times 25) / 2}=\frac{1 \times(25 / 2)}{2 \times 4 \times(25 / 2)}=\frac{12.5}{100}=
$$

$12.5 \%$, so the percent of the original 1 by 1 square which is shaded is $100 \%+12.5 \%=112.5 \%$.


$$
\frac{9}{8}=1 \frac{1}{8}
$$



$$
1+\frac{1}{8}=1 \frac{1}{8}
$$

## Part 2 - Dividing Fractions with Fraction Tiles

Many students have difficulty understanding what is being done when dividing by a fraction. It can be explained as finding how many "divisors" are in the "dividend." Using Exercise 4 as an example, they are trying to figure out how many $1 / 3$ 's are in 2 . Using the fraction tiles should help students visualize this concept.
4. Divide $2 \div \frac{1}{3}$. Answer: 6

Students will use both the TI-84 Plus and fraction tiles to show division of fractions.

To indicate division, students should use the $\ddagger$ key. For the example to the right, the key press is $2 \square$ ALPHA [F1] ENTER 1 - 3 ENTER.

Some students may experiment and figure out that they can use the fraction template to put the 2 in the numerator and then use the in the denominator to enter $1 / 3$.
5. Kara has $\frac{1}{2}$ cup of almond milk left. She uses $\frac{1}{4}$ cup each morning with breakfast. How many breakfast servings does Kara have left? Answer: 2
Dividing a fraction by a fraction, $\frac{1}{2} \div \frac{1}{4}$, adds a little complexity, but students should be familiar enough with the fraction template to perform this calculation. The key presses are ALPHA [F1] ENTER $1 \rightarrow 2$ ALPHA [F1] ENTER 1 - 4 ENTER.

Students may experiment and figure out that they can use fraction templates within fraction templates to create a stacked fraction.

6. Write a story problem in which the number 4 is divided by the fraction $\frac{1}{5}$.

Use the context of the problem to explain the relationship between multiplication and division.
Answer: One possible story: If a board is 4 meters in length, how many pieces can be cut if each piece must be $1 / 5$ meter?
Since $\frac{1}{5}$ meter multiplied by 20 is 4 meters, then 4 meters divided by $\frac{1}{5}$ meter is 20 .

| 1 |  |  |  | 1 |  |  |  |  | 1 |  |  |  |  | 1 |  |  |  |  |  |
| :---: | ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | ---: | ---: |
| $\frac{1}{5}$ | $\frac{1}{5}$ | $\frac{1}{5}$ | $\frac{1}{5}$ | $\frac{1}{5}$ | $\frac{1}{5}$ | $\frac{1}{5}$ | $\frac{1}{5}$ | $\frac{1}{5}$ | $\frac{1}{5}$ | $\frac{1}{5}$ | $\frac{1}{5}$ | $\frac{1}{5}$ | $\frac{1}{5}$ | $\frac{1}{5}$ | $\frac{1}{5}$ | $\frac{1}{5}$ | $\frac{1}{5}$ | $\frac{1}{5}$ | $\frac{1}{5}$ |

## Visualizing Fractions

## Part 3 - Multiplying and Dividing Decimals

7. $26.45 \times 0.25$ Answer: 6.6125

Students will likely be familiar with the key presses for multiplying and dividing on the home screen. Emphasize in this section that students should estimate the answer prior to performing the calculation as it is very easy to miss the decimal point or put it in the wrong location. Estimate answers may vary, but could include since $x 0.25$ means a quarter of the first number and 26.45 is
 close to 28 , then $28 / 4=7$. The answer will be close to but less than 7.

The key presses for the example at the right are $26 \square$


In the next three examples, take the opportunity to ask the students to round their answers to a particular decimal place.
8. $35.5 \div 4.2$ Answer: 8.452 (rounded to the nearest thousandth)

Estimate can be found by dividing 36 by 4 . Answer will be about 9 .

The key presses for the example at the right are $35 \square$ 5 5 [ 2 ENTER.
9. $14.25 \times 1.0825$

Answer: 15.426 (rounded to the nearest thousandth)
Estimate that a number a little bigger than 14 is multiplied by a number a little larger than 1 , so the answer will be close to 14 , but probably more like 15 .
10. $325 \div 18.25$

Answer: 17.808 (rounded to the nearest thousandth)


Estimate 18.25 as 20 and the problem becomes similar to dividing by two. 320/20=16.

These two exercises give students a chance to practice the keystrokes covered in Questions 7 and 8.

Enter the following keystrokes to find the solutions.

 ENTER.

Exercises 11 and 12 should be done without a calculator.
Have students justify their reasoning.
11. Answer: Choice C. 23.64
$9.85 \times 2.4$ can be estimated by finding $10 \times 2=20$.
12. Answer: Choice B. 95.5
$5006.11 / 52.42$ can be estimated by finding $\frac{5000}{50}=100$.
After students have completed the exercises, show the solutions to the class using the TI -SmartView ${ }^{\mathrm{TM}}$ emulator.
Key presses for the examples to the right are

$$
\begin{aligned}
& \text { 9 } 85 \text { ® } 2 \square 4 \text { ENTER and }
\end{aligned}
$$

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| 9.85*2.4 |  |
| :---: | :---: |
| รัø̈6.11<52.42 |  |
|  |  |



## Part 4 - Solve Similarity Problems

13. When Jeffrey was born, his parents planted a tree in the backyard. He has decided on his $13^{\text {th }}$ birthday to see how tall the tree is. Use proportions to determine the tree height. Show your work.

Answer: 180 inches or 15 feet.
Students can solve this problem a variety of ways. To find the height of the tree, $T$, set up a proportion:

$$
\frac{60}{40}=\frac{T}{120} \text { or } \frac{3}{2}=\frac{T}{120} .
$$

The tip of Jeffrey's shadow and the tip of tree's shadow coincide here.

Multiply the total shadow length, 120 , by $\frac{3}{2}$ to get a height for the tree of $T=180$ inches, or 15 feet.

Key presses for the example to the right are 100 खALPHA [F1] ENTER 3 -2 ENTER. To convert from inches to feet press $\square 10$ ENTER.

Students may also use a scale factor of 3. Tripling 40 will give 120. Therefore the height of the tree can be found by tripling 60, or 180 inches.

14. Moriah is 64 inches tall and casts a shadow that is 24 inches long. She is standing next to a billboard that casts a shadow 15 feet long. Use proportions to determine the distance from the ground to the top of the billboard. Show your work.

Answer: 40 feet.


It can be helpful to sketch two similar triangles.

To find the height of the billboard, $B$, set up a proportion.
$\frac{64}{24}=\frac{B}{15}$ or $\frac{8}{3}=\frac{B}{15}$.
Multiply the total shadow length, 15 , by $\frac{8}{3}$ to get a


24"
 height for the billboard of $B=40$ feet.

Key presses are 10 [5 ALPHA [F1] ENTER 8 - 3 ENTER.

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15* $\frac{8}{3}$
40.

