Induction for Whole Numbers

Student Activity

7 8 9 10 **11** 12

TI-Nspire™ Investigation Student 50 min

Introduction

The purpose of this activity is to use exploration and observation to establish a rule for the sum of the first n whole numbers then use proof by induction to show that the rule is true for all whole numbers.

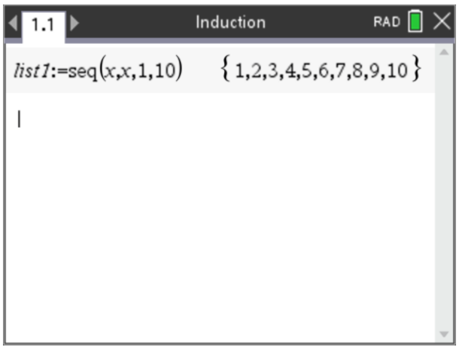
Calculator Instructions

The sequence command can be used to generate the first 10 whole numbers. These values could be entered directly into a list, however it is often handy to know where and how to use some of the calculator's commands so that when longer lists need to be generated they don't have to be entered individually.

[Menu] > **Statistics** > **List Operations** > **Sequence**

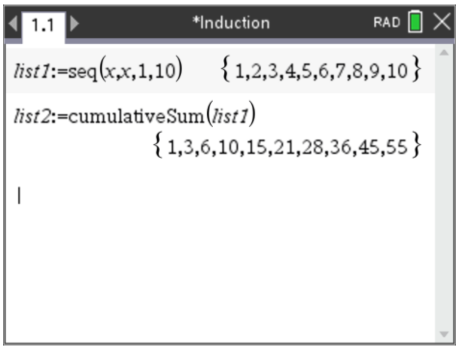
The sequence command can also be retrieved from the catalogue or typed directly. Store the numbers in List.

Syntax: seq(Expression, Variable, Start, End ,[Step])



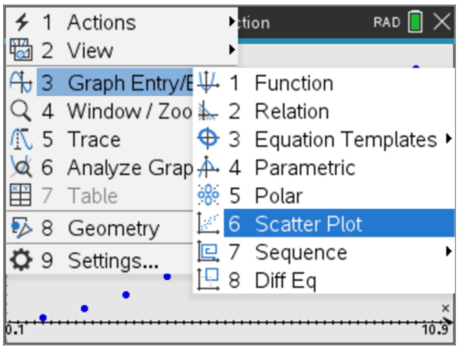
The cumulative sum of these numbers can also be computed. The instruction "cumsum()" can be typed directly or access from the List Operations menu.

Store this cumulative sum in a different list.



Insert a Graphs Application and set up a scatter plot to graph the points where List1 is plotted on the x axis and List2 on the y axis.

Make sure no other graphs are being plotted; then zoom in on the data.



Question: 1.

With reference to a difference table, explain why the relationship must be quadratic.

Question: 2.

Use simultaneous equations to establish values for a, b and c where the sum (s) can be expressed in the form: $s = ax^2 + bx + c$.

Question: 3.

Graph your equation to check that it passes through the points that have been plotted. Use $Y_1(x)$ and substitute a range of values to check your answer.

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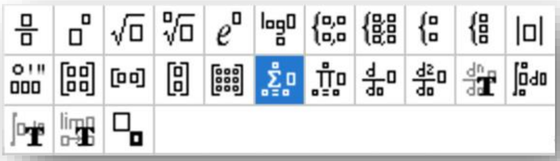
Question: 4.

Use your formula to determine the sum of the first 50 whole numbers.

Question: 5.

There is a summation template on the calculator. Use this template to determine the sum of the first 100 whole numbers and compare the result with your equation.

Notation: $\sum_{n=1}^{100} n$.



Pascal's + Triangle = Hidden Gem

The sum of the first n whole numbers is also referred to as Triangular Numbers. It is a lovely synchronism that Pascal's Triangle contain the triangular numbers.

The n^{th} triangular number is the third element in the $(n+1)^{th}$ row[1].

Example: The number 15 is the 5^{th} triangular number, it is the third element in the 6^{th} row.

Recall that the elements in Pascal's triangle can be computed using combinatorics: ${}^{n}C_r = \dfrac{n!}{(n-r)!r!}$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1

Generate the numbers: {2, 3, 4 … 12} and store the results in L_1.

A list of values can be generated from the combinatorics command, example: ${}^{L_1}C_2$.

Use this to generate the triangular numbers.

Question: 6.

a) Use your calculator and the combinatorics command to write down the first 10 triangular numbers. [Store the values in L_2]

b) Use combinatorics to calculate the 100^{th} triangular number, the sum of the first 100 whole numbers.

c) The diagonal for the triangular numbers in Pascal's Triangle can be written using combinatorics:

$${}^{n+1}C_2 = \frac{(n+1)!}{((n+1)-2)!2!}$$

Simplify this formula to write an expression for the n^{th} triangular number.

Question: 7.

Use induction (outlined below) to prove that $\sum_{n=1}^{x} n = \dfrac{x(x+1)}{2}$ is true for all positive whole numbers.

a) Show that your rule is true for $x = 1$

b) Assume the result for $\sum_{n=1}^{x} n = \dfrac{x(x+1)}{2}$ is true and show the rule holds for $x + 1$

[1] Row numbering in Pascal's triangle starts at row(0) = {1}, row(1) = {1, 1}, row(2) = {1, 2, 1}

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Question: 8.

The sum of the first x odd numbers can be expressed as: $\sum_{n=1}^{x}(2n-1)=x^2$.

a) Use your calculator to check that this equation is true for the first 10 odd numbers.

b) Use proof by mathematical induction to show that this rule is true for all odd numbers.

Question: 9.

The sum of the first x even numbers can be expressed as: $\sum_{n=1}^{x}(2n)=x^2+x$.

Use proof by mathematical induction to show that this rule is true for all even numbers.

Question: 10.

Use your calculator to determine a rule for the sum of the multiples of three: {3, 6, 9, 12 …}, then prove your result by mathematical induction. [Hint: Question 9 was a rule for even numbers, multiples of 2]

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