## REAL LIFE REAL WORLD Activity: Air Traffic Control

Topic: Angles of Depression and Trigonometry
Grade Level: 7-12
Objective: To model and solve air traffic control problems with geometry software. Use Cabri Jr, to construct a model of the problem situation and concepts from trigonometry and proportions to find the solution.

Time: 30-60 minutes

## Introduction

We can use a right triangle to model the approach of an airplane to an airport runway. Help the students come up with the triangle model. How would we represent the altitude of the plane? The angle of depression? Where would the airport be? What else do we know?

## Discuss with Students

Proportions can help us model the problem situation. A scale model is a proportion.

1. Suppose a map scale says 1 inch $=100$ miles. What would 5 inches represent? 2.5 inches?
2. Cabri Jr. measures using centimeter units. Using a scale of $1 \mathrm{~cm}=1000 \mathrm{ft}$, what distance is represented by the segment in Figure $1 ?$


Figure 1

## Discuss With Student Answers

1. 500 miles; 250 miles
2. $3,400 \mathrm{ft}$.

Student Page Answers
Activity 1
5.

Activity 2
5.

## Technology Reference

This activity uses the following Cabri Jr. functions:

| F2 | Point <br> Line | F3 | Perpendicular <br> Parallel | F5 |
| :--- | :--- | :--- | :--- | :--- | | Hide/Show |
| :--- |
| Alph-Num |

Refer to "Getting Started with Cabri Jr." for more details.
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Angles of depression (and elevation) are very important concepts. Although computers eliminate much of the calculation needed for trajectories, having a good feel for the mathematics involved provides a helpful backup. In this lesson, you will see important relationship between angles of depression and distance.

## Activity

1. Construct a right triangle: Construct a segment and a perpendicular line at one endpoint of the segment. Construct a right triangle using the endpoints of the segment and any third vertex on the perpendicular line.

- Press F2 and select the Segment tool. Draw a horizontal segment.

- Press F3 and select the Perp. tool. Move the cursor to the right endpoint and press ENTER to select it. The cursor now changes to an arrow to select the segment. Press ENTER to draw the perpendicular line.

- Press F2 and select the Triangle tool. Draw a right triangle by selecting the segment endpoints and any third vertex on the perpendicular line. Press F5: Hide/Show and select Object. Press ENTER and move the arrow to the perpendicular to select it. Press Enter and move away from the construction to watch the perpendicular disappear!


2. Use the Alph-Num tool to label the vertices as shown in Figure 8. The plane is located at point $P$, the airport at point $A$, and the altitude of the plane is indicated by the segment HA.

3. Find the measure of the angle of depression at point $P$, or angle APH. Press GRAPH for F5 and move to Measure and Angle. Press ENTER. You will now select 3 points to determine the angle. Select A, P, then H, by moving the cursor to each point and pressing the ENTER key. After selecting the third point, H , the angle measure will appear with the hand. Move it to a convenient location away from the angle. Press CLEAR to turn off the hand. Press CLEAR again to exit the angle measure tool.

4. Find the lengths of the segments PH and HA. Press GRAPH for F5 and move down to Measure and right to D. \& Length. Press ENTER. Move the pencil to an endpoint. When it is flashing, press ENTER. Move to the other endpoint and do the same thing. The length of the segment is shown. The hand indicates that it can be moved. Move it away from the segment to a convenient location. Press CLEAR to turn off the hand.

5. Note the segment lengths. Suppose we used a scale of $1 \mathrm{~cm}=1000$, how far is the plane, (represented by point $P$ ) from the airport (Point $A$ )? What is the altitude of the plane? What is the angle of depression?
6. Write a real-world scenario based on the model we have constructed
7. Suppose we change the model by dragging a vertex. Place the cursor near point $P$, which begin to flash. Press the [ALPHA] key and move $P$ nearer or farther from $A$. How does the scenario change? Change HA.
8. Design models to solve these problems:
a. If a plane flying at 2500 feet requires an angle of depression of 20 degrees to land at an airport, how far is the plane from the airport?
b. What angle of depression is needed for a plane 8600 feet from the airport if its current altitude is 4200 feet?
c. Will a plane at an altitude of 3 miles be able to land safely at an airport 5 miles away if its angle of depression is 15 degrees? Justify your answer.
9. What seems to be the relationship between angles of depression and distance?

A small angle of depression allows a very slow descent over great distances, while a larger angle of depression becomes necessary if a pilot needs to land a plane quickly.

## Extensions \& Resources

## Extension 1

Verify results of the problems from Activity 1 using trigonometric ratios.

## Extension 2

Find scenarios on the Internet to model on your calculator!

