How Random!

By Lynne Plettenberg

Time required
90 minutes

ACTIVITIES eXCHANGE **ID: 8555**

Activity Overview

In this activity, students use simulations and graphs to explore the commonsense notion that repeatedly flipping a coin results in "heads up" about half of the time. First, students simulate an experiment by representing single coin flips with random numbers. Next, they use a given formula to simulate multiple coin flips at once, allowing them to perform a large number of trials easily. Last, they create a histogram of their data, observing that the data generally vary around the mean.

Concepts

- *Simulations*
- *Binomial experiments*
- *Law of Large Numbers*

Teacher Preparation

This activity is designed for use in an Algebra 2 or Statistics classroom. It is best used to introduce the concept of data varying about the mean, which sets up a framework for future explorations of normal distributions.

- *Prior to beginning the activity, students should know how to interpret scatter plots and histograms and be familiar with binomial probability and the concept of mean as expected value.*
- *The screenshots on pages 84 and 85 (top) demonstrate expected student results. Refer to the screenshots on pages 85 (bottom) and 86 for a preview of the student TI-Nspire document (.tns file).*
- ***To download the student .tns files, go to education.ti.com/exchange and enter "8555" in the quick search box.***

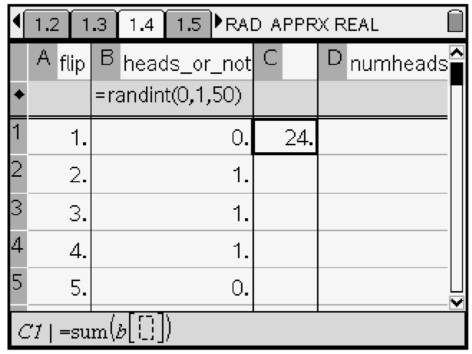
Classroom Management

- *This activity is intended to have students explore* ***individually and in pairs*** *(with the teacher providing assistance when needed). However, an alternate approach would be to use the activity in a whole-class format. By using the computer software and the questions found in the student .tns file, you can lead an interactive class discussion on simulating coin flips.*
- *For new users of the handheld, it would be beneficial to demonstrate how to select cells and columns in a spreadsheet, select and use commands from the Catalog (or enter them directly), and use data lists to create scatter plots and histograms in the Graphs & Geometry and Data & Statistics applications.*
- *The TI-Nspire solution document StatAct04_HowRandom_EN.tns shows the expected results of working through the activity.*

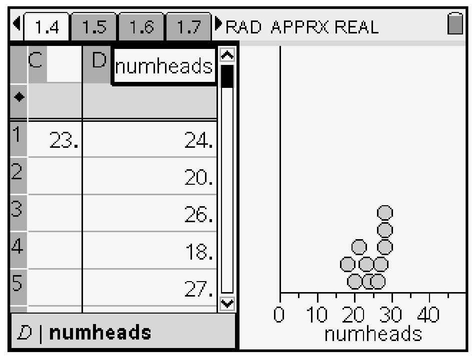
TI-Nspire™ Applications

Graphs & Geometry, Lists & Spreadsheet, Notes, Data & Statistics

Problem 1 – How many heads to expect?



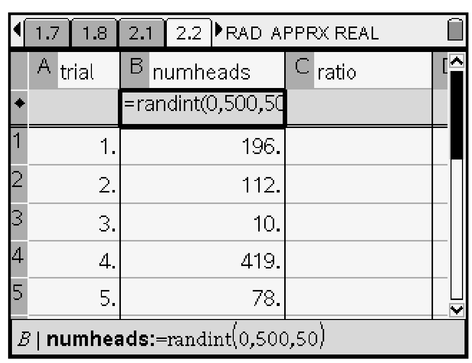
On page 1.4, students first need to select cells A1 and A2. To do so, position the cursor on cell A1, press and hold ⇧, and arrow down to extend the selection. Then they can use the **Fill Down** command from the Data menu to place the numbers 1-50 in Column A, representing the flips. For Column B, guide students as needed (filling in the arguments) to write the formula **=RandInt(0,1,50)** in the gray formula cell to generate a list of 50 random 0s (tails) and 1s (heads). Discuss how this models the situation.



Next, students are prompted to enter the formula **=sum(b[ ])** in cell C1 to return the total number of heads for the 50 flips, as shown. Recording the calculated value of the sum in cell D1, students should then "repeat the experiment" by pressing ctrl + R (**Recalculate**) to obtain a different set of 0s and 1s. The sum in cell C1 will update as the random numbers change; students should record at least 10 such sums in Column D. Rather than having to scroll up to cell C1 to find sums 6 through 10, students can reenter **=sum(b[ ])** in cell C6.

Once 10 sums are recorded, have students select Column D (by arrowing all the way to the top of the column, or selecting **Select** > **Select Column** from the Actions menu) and choose **MENU > Data > Quick Graph**, which displays a dot plot of the data. Setting the Xmin at 0 and the Xmax at 50, students will find that most of the data points fall around 25.

Problem 2 – Even more coin flips

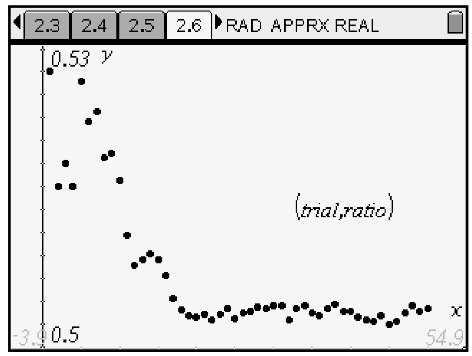


In this problem, students are presented with a more complicated situation. This time, they will simulate flipping 500 coins, 50 times! It may appear that the formula to use to generate values for each trial is as simple as randomly choosing 50 numbers from 0 to 500. Students are prompted to use this formula, **=RandInt(0,500,50)**, to generate their trials. It should be immediately obvious to students that this formula is not correct—based on the results of Problem 1, the number of heads should generally vary around 250. This is not the case, as these data are evenly distributed across the interval 0 to 500. Guide students to enter the correct formula **=sum(randInt(0,1,500))** in cell B1 and **Fill Down** to populate the other trials. (***Note:*** The formula must be entered in the cells directly; not in the formula cell.)

Explain how the formula **=cumSum(b[])/(500*a[])** gives the cumulative ratio of the total number of heads to the total number of flips, for each trial. Ask students to predict what value they expect this ratio to approach as more and more trials are considered. The formula may then be entered in the formula cell for Column C to populate the column.

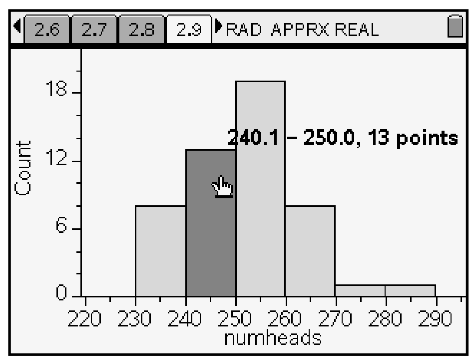
The scatter plot *trial vs. ratio* is displayed on page 2.6. While each student's scatter plot is unique, all should find that the data points generally fall along the horizontal line $y = 0.5$.

Selecting **MENU > Window > Zoom – Data** provides a closer view of the data, where students are asked to describe what happens as the number of trials increases (the ratio converges to 0.5).



Finally, students make a histogram of the **numheads** data. To do so, click below the *x*-axis, select **numheads** as the variable, and choose **Histogram** from the Plot Type menu. The majority of the data should fall around 250, as expected. Adjusting the window to see slightly beyond the data, students can grab and drag a shared side of a bar to change the width of the intervals, and clicking on a bar will display the interval it represents and the number of data values in that interval. (***Note:*** Caution students not to drag the bar itself—doing so would change the data!)

The mean of the data set is roughly 250, and the standard deviation is about 10. Have students drag one side of a bar to 250 and the other side of the same bar to 260. This sets the width of the bars at 10. By clicking on bars and summing the counts, students are then asked to find the *percent* of values that fall within 240–260 and 230–270—or one and two standard deviations from the mean. This helps to build a foundation for understanding the empirical rule of normal distributions.



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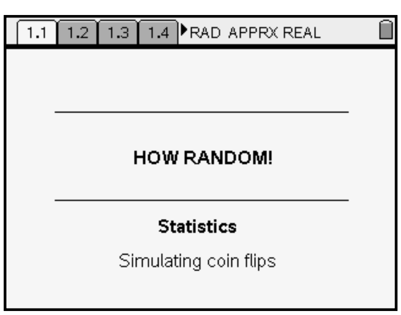
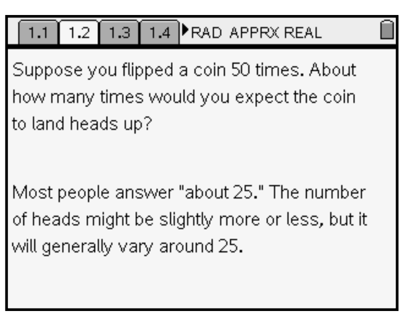
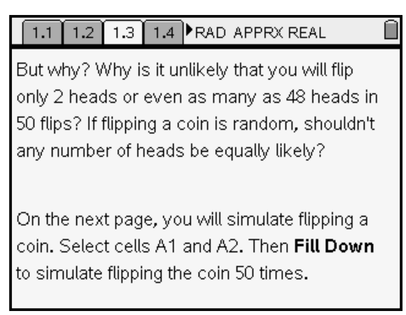






1.1 1.2 1.3 1.4 RAD APPRX REAL

	A flip	B heads_or_not	C	D numheads
◆				
1	1.			
2	2.			
3				
4				
5				

A1 |

1.2 1.3 1.4 1.5 RAD APPRX REAL

Next, we need to simulate "getting heads or tails." In the formula cell for Column B, use the **randInt(** command to generate a list of 50 values: 0s will represent tails, and 1s will represent heads.

1.3 1.4 1.5 1.6 RAD APPRX REAL

Think about a formula you could use to quickly calculate the total number of heads from those 50 flips.

Enter the formula for the sum in cell C1.

Record the calculated value in cell D1.

1.4 1.5 1.6 1.7 RAD APPRX REAL

Now we need to repeat the experiment a few more times. Press **CTRL+R** to randomly repopulate Column B with 1s and 0s.

Repeat for a total of 10 trials, keeping a record of each sum by entering it in the next empty cell in Column D. (You can reenter the formula for the sum in cell C6 so you don't have to scroll up and down for trials 6–10.)

1.5 1.6 1.7 1.8 RAD APPRX REAL

Select Column D and make a **Quick Graph** of the data. Adjust the **Window Settings:** Set Xmin and Xmax at 0 and 50, respectively.

You should find that the data points fall around the "middle," or 25. If they do not, it may be that not enough trials were performed for a pattern to appear. We will explore even more trials in the next problem.

1.6 1.7 1.8 2.1 RAD APPRX REAL

Now suppose instead of flipping the coin 50 times, you flip it *500 times*. And instead of performing 10 trials, you perform *50 trials*! How many heads would you expect to flip on each trial?

Do you think the formula **randInt(0,500,50)** would make sense to simulate this situation? Why or why not? Try it and see.

1.7 1.8 2.1 2.2 RAD APPRX REAL

	A trial	B numheads	C ratio
◆			
1	1.		
2	2.		
3	3.		
4	4.		
5	5.		

A1 |

1.8 2.1 2.2 2.3 RAD APPRX REAL

Go back and look closely at your data. Of the 50 values, how many are under 200?

Explain why this does not make sense.

Think about how you might use the **sum(** *and* **randInt(** commands to correctly represent 50 random numbers of heads out of 500 flips.

2.1 2.2 2.3 2.4 RAD APPRX REAL

Return to the spreadsheet, and clear the formula cell for Column B. Enter your formula in cell B1 (remember to include an equals sign), and **Fill Down** from B1 to populate all 50 trials.

Do these data values make more sense?

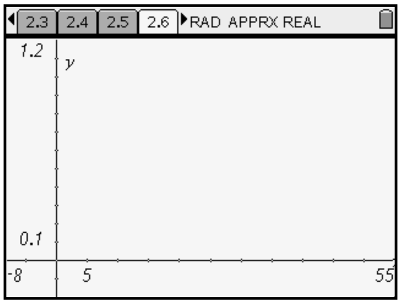
2.2 2.3 2.4 2.5 RAD APPRX REAL

Now, for each trial, we'll look at the cumulative ratio of the total number of heads to the total number of flips.

Enter **=cumSum(b[ ]) / (500*a[ ])** into the formula cell for Column C. Predict the value of this ratio as more trials are considered.

The scatter plot *trial vs. ratio* is displayed on the next page.

2.3 2.4 2.5 2.6 RAD APPRX REAL



2.4 2.5 2.6 2.7 RAD APPRX REAL

What do you notice about the scatter plot?

Do the points fill the viewing window?

Select **Zoom – Data** from the Window menu to gain a closer view of the data.

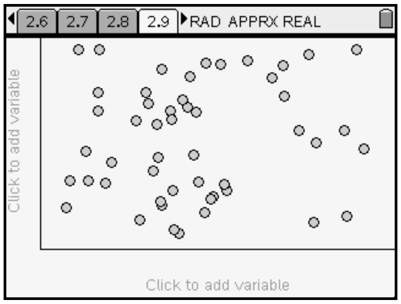
Describe what happens as the number of trials increases.

2.5 2.6 2.7 2.8 RAD APPRX REAL

Now we'll take a different look at the data. On the next page, create a histogram displaying the data in Column B. Click below the *x*-axis, choose **numheads**, and select **Histogram** from the Plot Type menu.

Adjust the window so you can see a little past each of the "bookend" bars and above the highest bar.

2.6 2.7 2.8 2.9 RAD APPRX REAL



2.7 2.8 2.9 2.10 RAD APPRX REAL

You can grab and drag the side of a bar to change the width of the bars. Clicking on a bar will show its interval and the number of trials in that interval.

Theoretically, how many heads would you expect out of 500 flips?

What percent of these 50 trials resulted in 240–260 heads? How about 230–270 heads?