Volume by Cross Sections

ID: 12280

Time Required
15 minutes

Activity Overview

In this activity, students will be introduced to the concept of finding the volume of a solid formed by cross sections of a function that form certain shapes. Since volume is the area of the base times the height and dV = Area·dx, students review areas of various shapes like squares, semicircles and equilateral triangles. Calculator screenshots are used to help students get a visual of the volume under consideration. Students will practice what they learn with exam-like questions.

Topic: Volume by Cross Sections

- *Applications of integration*
- *Volume by cross sections*

Teacher Preparation and Notes

- *Part 1 of this activity takes less than 15 minutes. Part 2 contains three exam-like questions that have accompanying visuals that can be used as an extension or homework.*
- *Students will write their responses on the accompanying handout where space is provided for students to show work when applicable.*
- ***To download the student worksheet, go to education.ti.com/exchange and enter "12280" in the keyword search box.***

Associated Materials

- *VolumeCrossSection_Student.doc*

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *The Region Between Two Curves (TI-89 Titanium) — 3464*

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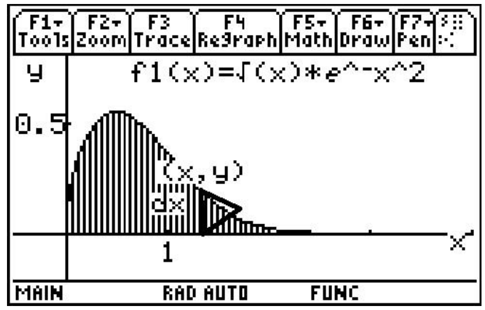
Part 1 – Setting Up The Problem And Understanding The Concept

In this section students are introduced to the concept of finding the volume of a solid formed by cross sections of a function that form certain shapes. Since volume is the area of the base times the height and dV = Area dx, students review areas of various shapes like squares, semicircles and equilateral triangles.



Part 1 ends with students finding the volume with equilateral triangle cross sections.

Student Solutions

1. dx
2. **a.** base times height. The area of a square with side x is x^2.

   b. $\frac{1}{2}\pi r^2$
3. $\frac{1}{2}y\frac{\sqrt{3}}{2}y$
4. 0.433013 cm^2
5. $\int_0^2 \frac{1}{2}y\frac{\sqrt{3}}{2}y\ dx = \int_0^2 \frac{1}{2}\left(\sqrt{x}\cdot e^{-x^2}\right)\frac{\sqrt{3}}{2}\left(\sqrt{x}\cdot e^{-x^2}\right)dx$

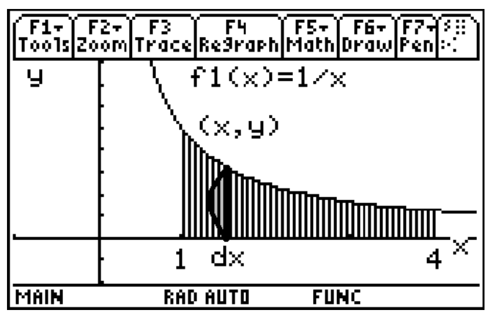
   $= \int_0^2 \frac{\sqrt{3}}{4}x\cdot e^{-2x^2}dx$

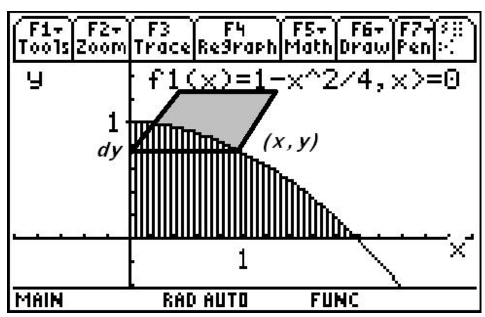
   If students use u-substitution, $u = -2x^2$, $du = -4x\ dx$ and the limits of integration are from 0 to –8.

   $-\frac{\sqrt{3}}{16}\int_0^{-8} e^u du = -\frac{\sqrt{3}}{16}\left(e^{-8}-1\right) = \frac{\sqrt{3}}{16}\left(1-\frac{1}{e^8}\right)$

Part 2 – Homework

This section enables students to get a visual of challenging exam-like questions. Students should show their work on the first two questions and show their set up on the third question.





Student Solutions

1. $\frac{3\pi}{32}$ units³
2. 2 units³
3. 1.57 units³