Radio Station KTNS

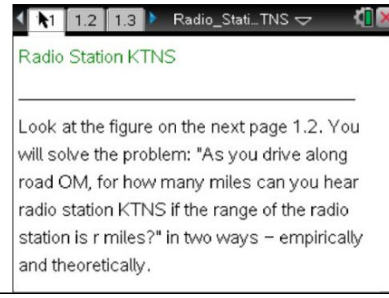
Student Activity

Name ______________

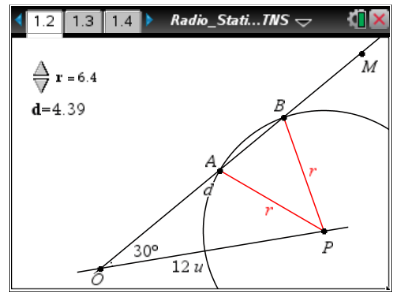
Class ______________

Open the TI-Nspire document Radio_*Station_KTNS.tns.*

In this activity, you will explore a real-world example of the Law of Sines and the Law of Cosines.



Radio Station KTNS is located at point P in the figure. The range of its signal is r miles, meaning that people within r miles of P would be able to hear the station. You are driving along road OM at an angle of 30° with OP. For how many miles, d, could you hear station KTNS?



In ΔPAB, the Law of Cosines tells us that $d^2 = 2r^2 - 2r^2 \cdot \cos(\angle APB)$, so it is reasonable to assume that d^2 could be a linear function of r^2. To solve this problem, you will determine d^2 in terms of r^2 in two ways:

- Find an experimental model by gathering data and fitting an appropriate regression function to the data.
- Find a theoretical model using the Law of Sines, the Law of Cosines, and algebra.

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Press ctrl ▶ and ctrl ◀ to navigate through the lesson.

The figure is a scale drawing with 1 unit = 10 miles so that OP = 12 units or 120 miles.

1. In miles, the reasonable values of r satisfy $k < r \leq 120$. What is the value of k? Why?

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Using the slider, the following data has been gathered in the spreadsheet in the four columns: $rad(r)$ $dis(d)$ $r2 = r^2$ $d2 = d^2$

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A scatterplot of the data has been drawn on this page.

Move to page 1.5.

2. Fit a linear regression function to the data with $x = r2$ and $y = d2$ in units. Select **MENU > Statistics > Stat Calculations > Linear Regression (mx+b)**. with $r2$ for X List, $d2$ for Y List, and **Save RegEqn** to: $f1$.

   Record your answer here: $d^2 =$ _____ $r^2 -$ ______.

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3. Plot the regression equation on the scatterplot, and note how well it fits. Open the entry line, move back up to $f1(x)$, and press [enter].

   According to this linear model, for how many miles, d, could you hear the station if $r = 90$ miles?
   Hint: Remember r = 9 units corresponds to r = 90 miles.

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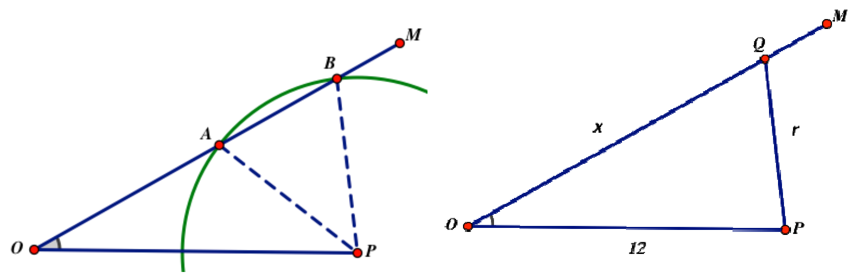
Theoretical Model

Find the theoretical function expressing d^2 in terms of r^2 by completing the argument below.



4. The figure for this problem shows an example of an ambiguous case of the Law of Sines since there are two triangles with two sides OP = 12, r, and the non-included angle of 30°. Consequently, if we apply the Law of Cosines to a triangle with sides OP = 12, r, x and angle 30°, we obtain the equation:

   ____________________________________ = 0.

On the scale drawing, then, the two solutions for x are OA and OB, and the distance, d, is $d = OB - OA$.

5. a. Find the two solutions for x of this equation.______________________________ .
   Hint: You can use "solve" command. Both solutions will be functions of r^2

b. Find the difference of the two solutions and express d^2 in terms of r^2 in units:

d^2 = ________________________

6. How does your theoretical equation compare to the regression equation?

7. According to this theoretical model, for how many miles, d, could you hear the station if $r = 90$ miles?
Hint: Remember r = 9 units corresponds to r = 90 miles.

8. Suppose the angle between the two roads OP and OM is changed to $\theta°$. Express d^2 in terms of r^2 and θ:

d^2 = ________________________

