Time Derivatives

ID: 9536

Time required
45 minutes

Topic: Application of Derivatives

- *Given the distance-time function for a moving object, calculate its velocity and acceleration at a given time.*
- *Given the distance-time function for a moving object, identify the intervals in which the object is moving at a constant speed, accelerating, or decelerating.*
- *Apply the chain rule to solve problems involving related rates.*
- *Use derivative to solve problems in models of exponential growth and decay.*

Activity Overview

In this activity, students will learn how to find velocity and acceleration and identify when the object is at rest, accelerating, and decelerating. In addition, the student will work with related rate problems, exponential growth, decay, and cooling problems.

Teacher Preparation

- *This investigation uses **fMax** to answer a question. Students will have to graph, take derivatives, and solve on their own.*
- *The calculator should be set in radian mode before trigonometric derivatives are taken.*
- *The screenshots on pages 1–4 demonstrate expected student results.*
- ***To download the student worksheet, go to education.ti.com/exchange and enter "9536" in the quick search box.***

Classroom Management

- *This activity is designed to be **student-centered** with the teacher acting as a facilitator while students work cooperatively. The student worksheet is intended to guide students through the main ideas of the activity and provide a place for them to record their observations.*
- *The students will need to be able to enter the functions and use the commands on their own.*
- *Before starting this activity, students should go to the home screen and select **F6:Clean Up > 2:NewProb**, then press [ENTER]. This will clear any stored variables, turn off any functions and plots, and clear the drawing and home screens.*

TI-89 Titanium Plus Applications

Graph

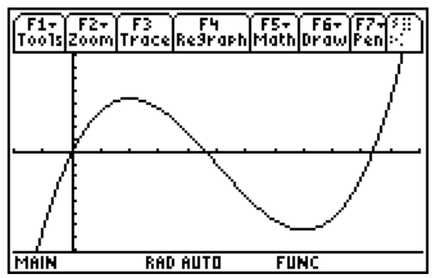
Introduction

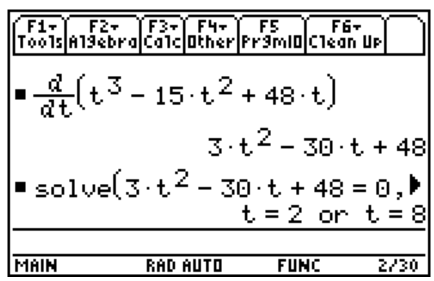
The main concept in this activity, time derivatives, relates to using time, t, as the basic underlying variable. If the function is a position function, then time, t, is the variable. **Velocity, $v(t)$,** is the time rate of change of distance so it is the first derivative of the position function. **Acceleration, $a(t)$,** is the time rate of change of the velocity (are we getting faster or slower?) so it is the first derivative of velocity as well as the second derivative of the position function.

If the function is a related rates exercise, a growth problem, decay problem, or cooling problem, then the function is a relationship among different variables but all the variables change with respect to time.

The distance or position function is directional. Hence moving right or up is positive and moving down or left is negative. Thus we can have a negative velocity (a ball is dropping or a car is heading west or south) or a positive velocity (a ball is thrown up in the air or a car is heading east or north).

Problem 2 – Velocity and acceleration of position functions





Student are to graph the function $s(t)=t^3 – 15t^2 + 48t$. When entering the function, they will need to use x in place of t. Students can use the **Derivative** command and **Solve** command in the Home screen to help find the answers.

- $v(t) = s'(t) = 3t^2 – 30t + 48$

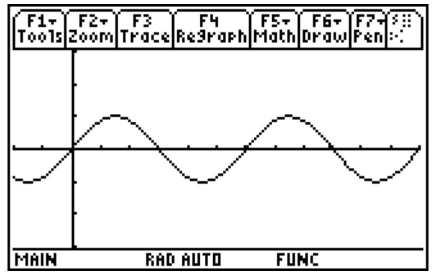
To find when the velocity is negative, positive and at rest, student will need to factor the velocity function.

- v(t) > 0 when t < 2 or t > 8
  v(t) < 0 when 2 < t < 8
  v(t) = 0 when t = 2, t = 8

To find when the acceleration is negative, positive, or constant, students will need to find the zeros of the acceleration function.

- $a(t) = v'(t) = s''(t) = 6t – 30$
- $a(t) > 0$ when $t > 5$
  $a(t) < 0$ when $t < 5$
  $a(t) = 0$ (constant speed) when $t = 5$

Students are to graph the function $s(t) = \sin\left(\frac{\pi \cdot t}{3}\right)$.



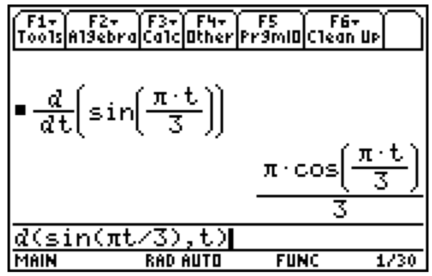
- $v(t) = s'(t) = \frac{\pi}{3}\cos\left(\frac{\pi t}{3}\right)$

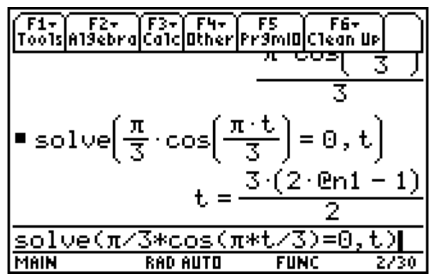
Since cos(t) > 0 in the interval $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$, then,

$\cos\left(\frac{\pi t}{3}\right) > 0$ on (–1.5, 1.5). If students use the

solve command with $\frac{\pi}{3}\cos\left(\frac{\pi t}{3}\right)$>0, they will get

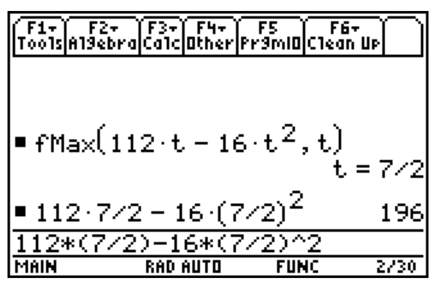
$(-1.5 \pm 6n, 1.5 \pm 6n)$ where n is an integer.





- $v(t) < 0$ when t is in $(1.5 \pm 6n, 4.5 \pm 6n)$
  $v(t) = 0$ when $t = 1.5 \pm 3n$.
- $a(t) = v'(t) = s''(t) = \frac{-\pi^2}{9}\sin\left(\frac{\pi t}{3}\right)$
- $a(t) > 0$ when t is in $(-3 \pm 6n, \pm 6n)$
  $a(t) < 0$ when t is in $(\pm 6n, 3 \pm 6n)$
  $a(t) = 0$ (constant speed) when $t = \pm 3n$

In this problem, the students can use the **fMax** command. Since the curve is a parabola oriented downward, they do not need to restrict the domain.



The value the students get is the time. They still have to evaluate that time in the function.
S(7/2) = 196 feet.

The ball lands back on the ground when t = 7. Students can use **solve** on the original equation to find that t = 0 when the ball was shot vertically and t = 7 when the ball slams into the ground.

Problem 2 – Use the chain rule in related rate problems

Since the rate the radius of the balloon is changing is $\frac{dr}{dt} = 3\frac{\text{cm}}{\text{min}}$, the equation for the radius at time t is $r = 3t$. Student can use this equation to find the time at which the radius of the balloon is 8cm (t = 2.67 minutes).

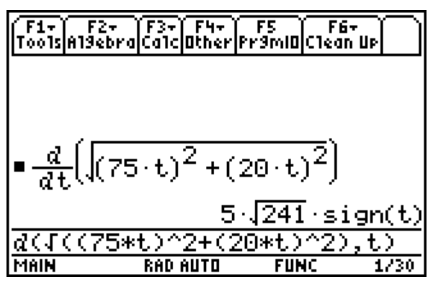
Students will then take the derivative of the volume with respect to time. $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$$\frac{dV}{dt} = 4\pi(8cm)^2\left(3\frac{cm}{\text{min}}\right) = 768\pi \text{ cubic cm/min}$$

The rate that the radius of the ripple is changing is $\frac{dr}{dt} = 40\frac{\text{cm}}{\text{s}}$. So the equation of the radius at t is $r = 40t$. The area of the ripple for any radius and its time derivative is $A(t) = \pi r^2 \rightarrow \frac{dA}{dt} = 2\pi r\frac{dr}{dt}$.

	t = 1	t = 3	t = 5
r	40 cm	120 cm	200 cm
$\frac{dA}{dt}$	$2\pi(40)(40) = 3200\pi\ \frac{\text{cm}^2}{\text{s}}$	$2\pi(120)(40) = 9600\pi\ \frac{\text{cm}^2}{\text{s}}$	$2\pi(200)(40) = 16000\pi\ \frac{\text{cm}^2}{\text{s}}$

From the problem, the rate of the car traveling east is $x = 75t$ and the rate of the car traveling north is $y = 20t$. Students will need to use the distance formula or Pythagorean theorem to determine function for the distance between the cars.



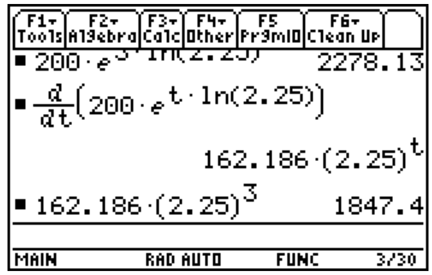
$$s(t) = \sqrt{x^2 + y^2} \rightarrow s(t) = \sqrt{(75t)^2 + (20t)^2}$$

So $s(t) = 5t\sqrt{241} \rightarrow s'(t) = 5\sqrt{241}$

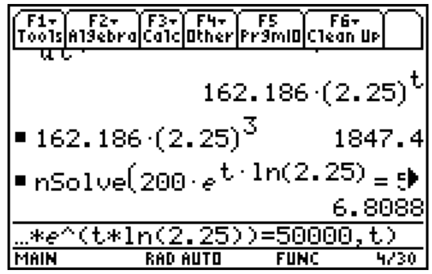
The sign(t) in the answer reflects the fact that the calculator doesn't know whether t is positive or negative and the answer should be positive.

Problem 3 – Growth and Decay Derivatives

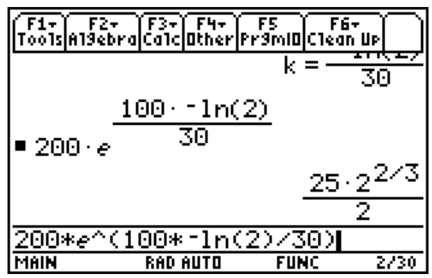
In the growth problem,



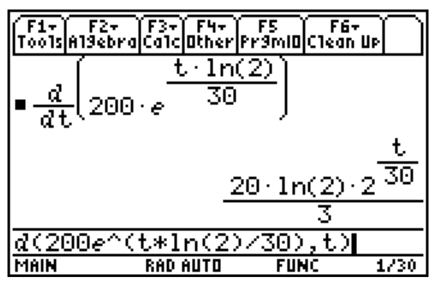
- $A(0) = 200$
  $A(1) = 450$
  $450 = 200e^{k} \rightarrow 2.25 = e^{k} \rightarrow \ln(2.25) = k$
- $A(t) = 200e^{t\ln(2.25)}$
  $A(3) = 200e^{(3)\ln(2.25)} = 2278.13$
- $A'(t) = \left(\ln(2.25)\right) \cdot 200e^{t\ln(2.25)}$
  $A'(3) = \left(\ln(2.25)\right) \cdot 200e^{(3)\ln(2.25)} = 1847.4$
- Students are to solve $A(t)$=50000. The **nsolve** command gives the decimal time rather than a fraction. (t = 6.8088 hrs)



In the decay problem,



- $A(0)$ =200,
  A(30) = 100 (half of $A(0)$)
  $100 = 200e^{30k} \rightarrow k = \dfrac{-\ln(2)}{30}$
- $A(t) = 200e^{-\frac{t \cdot \ln 2}{30}}$
  $A(100) = 200e^{-\frac{100 \cdot \ln 2}{30}} = 19.84$
- $A'(t) = \dfrac{-20 \cdot \ln(2) \cdot 2^{-\frac{t}{30}}}{3}$
  $A(100) = \dfrac{-20 \cdot \ln(2) \cdot 2^{-\frac{100}{30}}}{3} = -0.458$
- Students solve $1 = 200e^{-t\ln(2)}$ for t, **nsolve** gives 229.316



Extension – Cooling Derivatives

- $T(0)$ = 120, $T(30)$ = 100, $T(sur)$=70, so c = 50
  $100 = 70 + 50e^{30k} \rightarrow k = \dfrac{-\ln(5/3)}{30}$.
- $T = 70 + 50e^{\frac{-\ln(5/3)}{30}t}$
- Solve for t. It will take 135.2 minutes.