

Derivatives: Instantaneous Rate of Change

by

Mary Ann Connors

Department of Mathematics
Westfield State College
Westfield, MA 01086

Textbook Correlation: Key Topic

- Derivatives

NCTM Principles and Standards:

- Process Standard
 - Representation
 - Connections

Exercises and Solutions:

A. Intuitive Exploration of the Derivative of the Cosine Function.

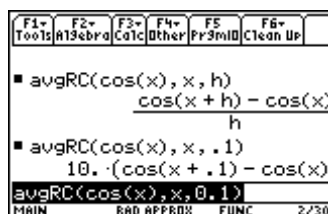
1. Estimate the instantaneous rate of change (derivative) of the cosine function by using the average rate of change of cosine with respect to x . Let h equal the change in x .

Solution:

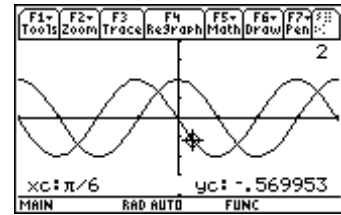
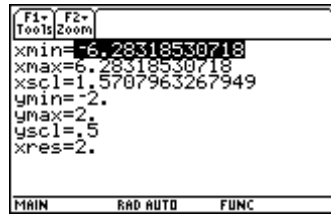
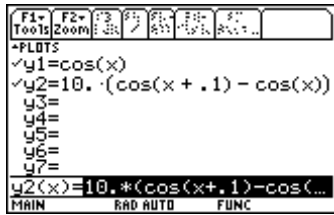
The average rate of change command (**avgRC**) is in the **CATALOG** of the TI-89.



Compute the average rate of change with step, h . Then let $h = 0.1$ and smaller values.

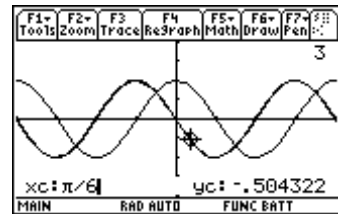
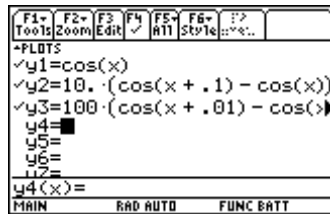


Highlight $y_1 = \cos(x)$ in the Y= editor. Select **F6(Style)**, **4:Thick**. Select **F6(Style)**, **1:Line** for y_2 . Recall the copy and paste feature. Copy the highlighted expression on the Home Screen and paste it in the Y= editor. Do you recognize the graph of $y_2(x)$?



Answer: $y_2(x)$ appears to be approximately equal to negative $\sin(x)$

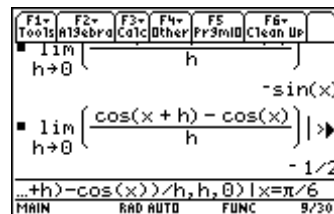
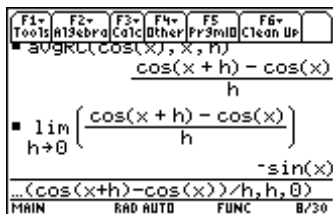
- Estimate the instantaneous rate of change (derivative) of $y = \cos(x)$ at $x = \pi/6$ using the average rate of change for increasingly small values of h .



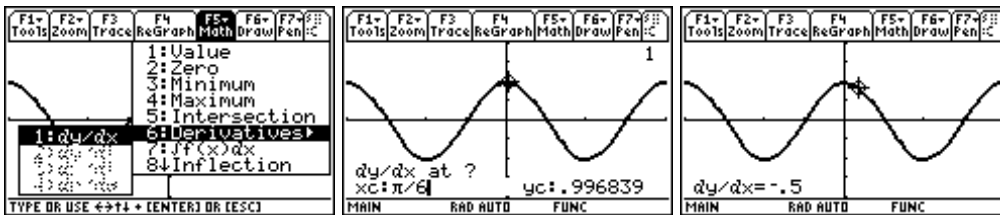
Answer: Approximately $-.5$

B. Using the Definition of the Derivative to Find the Answer Symbolically

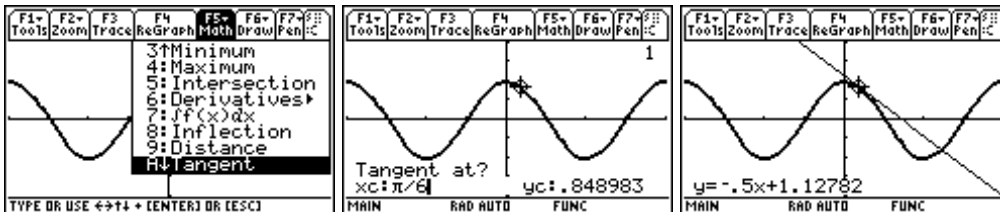
- Use the definition of the derivative to verify your observation in Part A.



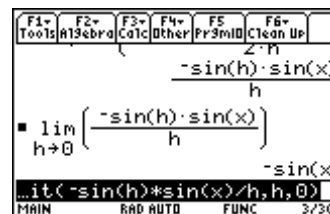
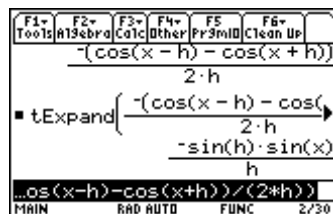
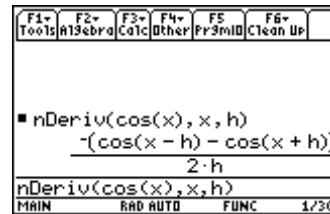
- Graph $y_1(x) = \cos(x)$. In the **GRAPH** window, press **F5(Math)**, **6:Derivatives**, **1:dy/dx**. Type in the value of $x (\pi/6)$. The derivative of $\cos(x)$ at $\pi/6$ appears in the lower left corner of the screen.



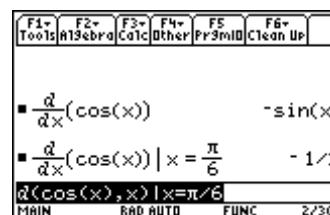
3. Draw the tangent line at $x = \pi / 6$. Follow the procedure pictured below. Notice the equation in the lower left corner of the screen. What is the slope of the tangent line?



4. Use **F3 (Calculus)**, **A:nDeriv(** on the Home screen to compute the numerical derivative using the symmetric difference quotient.



5. Evaluate exact derivatives (**2nd**, **8** for $d($ or **F3 (Calculus)**, **1:d (differentiate)** on the Home Screen.



Additional Exercises:

1. Use the average rate of change of $f(x) = 5^x$ for small values of h (.001, .0001, etc.) to estimate its instantaneous rate of change (derivative).
2. Evaluate its limit at $x = 0$ to find the instantaneous rate of change (derivative) at $x = 0$.
3. Repeat #1 and 2 for the function, $f(x) = x^{1/5}$.