Parametric Equations

Graphing calculators are used mostly for graphing equations in which y is a function of x; but y is not a function of x in a graph that's a circle or a polygon, because most values of x in the domain correspond to more than one value of y. Therefore to graph a circle or a polygon we use a third variable, usually called t, and write x as a function of t and also write y as a function of t. Students can think of t as giving a time and x and y as giving the positions of a bug crawling on the screen. These functions are called *parametric equations.*

[Link] Students will learn more about parametric equations and trigonometric functions in a later math course.

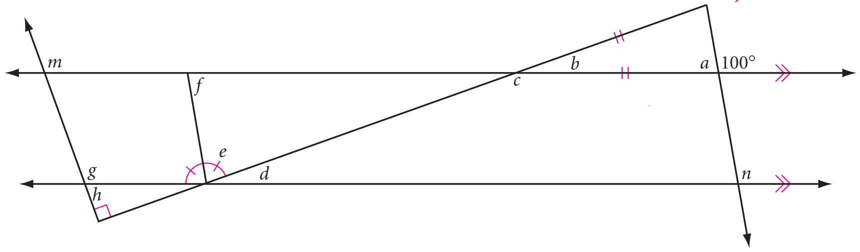
The focus of the project is on the central angles of a regular polygon. Help students keep the mathematical ideas in mind even while they are immersed in the details of graphing.

Friendly Windows

To help students understand friendly calculator windows, explain that *pixels* are dots on the screen. Pictures are made by turning selected pixels dark and leaving the rest light. If a window has 94 pixels across and you select the x-window to be $[-4.7, 4.7]$, then every pixel represents a distance across of $\frac{9.4}{94} = 0.1$ unit. Or, if a screen has 62 pixels vertically, you could select a y-window of $[-31, 31]$ to give a vertical value of 1 unit per pixel. If you set the window so that each pixel represents a different number vertically than horizontally, figures will appear distorted. Avoiding this distortion may not be important when you are graphing functions. But if you want a regular hexagon to look regular, you need to use a friendly window, that is, one in which each pixel represents the same number of units vertically as horizontally.

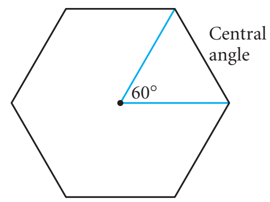
Review

17. Trace the figure below. Calculate the measure of each lettered angle. $a = 80°, b = 20°, c = 160°, d = 20°, e = 80°, f = 80°, g = 110°, h = 70°, m = 110°, n = 100°$



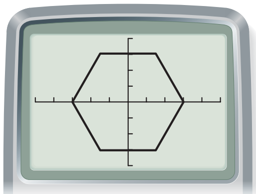
project

DRAWING REGULAR POLYGONS



You can draw a regular polygon's central angle by extending segments from the center of the polygon to its consecutive vertices. For example, the measure of each central angle of a hexagon is 60°.

Using central angles, you can draw regular polygons on a graphing calculator. This is done with parametric equations, which give the x- and y-coordinates of a point in terms of a third variable, or parameter, t.



Set your calculator's mode to degrees and parametric. Set a friendly window with an x-range of -4.7 to 4.7 and a y-range of -3.1 to 3.1. Set a t-range of 0 to 360, and t-step of 60. Enter the equations $x = 3\cos t$ and $y = 3\sin t$, and graph them. You should get a hexagon.

The equations you graphed are actually the parametric equations for a circle. By using a t-step of 60 for t-values from 0 to 360, you tell the calculator to compute only six points for the circle.

Use your calculator to investigate the following. Summarize your findings.

- Choose different t-steps to draw different regular polygons, such as an equilateral triangle, a square, a regular pentagon, and so on. What is the measure of each central angle of an n-gon?
- What happens as the measure of each central angle of a regular polygon decreases?
- What happens as you draw polygons with more and more sides?
- Experiment with rotating your polygons by choosing different t-min and t-max values. For example, set a t-range of -45 to 315, then draw a square.
- Find a way to draw star polygons on your calculator. Can you explain how this works?

Supporting the project

If the steps were infinitely small, the two parametric equations would generate a circle. Considering this case can help students see how the calculator is using the central angle to draw a polygon.

OUTCOMES

- The polygons are regular. (A friendly window was used.)
- The measure of the central angle of each polygon is given, and answers to the questions are clear.
- Rotated polygons are shown.
- Star polygons are created and explained.