





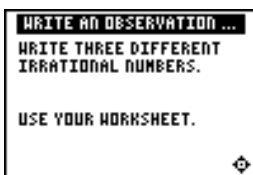
**Number Sense: Real Numbers** **Student Worksheet**

**Overview**

The **Overview** introduces the topics covered in **Observations** and **Activities**. Scroll through the **Overview** using  () to review, if necessary). Read each screen carefully. Look for new terms, definitions, and concepts.

**Observations**

The **Observations** illustrate number sense concepts relating to real numbers. Scroll through the **Observations** using  () to review, if necessary). Read each screen carefully. When you come to a **Write an Observation** screen, stop and write the answers to the questions on your worksheet.



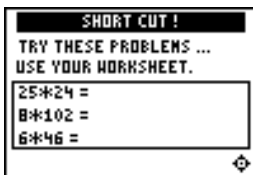
**Observation 1**

Write three different irrational numbers. Show your work.

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**Observation 2**

Try these problems.

Use the real number properties to solve the following problems quickly. Show your work.

$25 \times 24 =$  \_\_\_\_\_

$8 \times 102 =$  \_\_\_\_\_

$6 \times 46 =$  \_\_\_\_\_

## Activities

The **Activities** help you practice real number concepts. You can select from two activities—**Raining Reals** and **What Is My Property?** Follow these steps to play the activity and complete your worksheet.

1. Make sure you are in the **Activities** for this section.
2. Highlight an activity using  $\uparrow$  or  $\downarrow$ , and press **ENTER**.



**Scoring:** Every correct placement earns 2 points.

The game automatically ends if you have answered incorrectly four times (shown in the top right corner), or you press **(QUIT)** to stop.

### Raining Reals

1. Highlight a level (silver = less difficult; gold = more difficult), and press **ENTER** to select it.
2. As the numbers fall on your screen, quickly determine if the “raining” number is rational or irrational.
3. Press  $\leftarrow$  to move the number into the **RATIONAL** set, or press  $\rightarrow$  to move the number into the **IRRATIONAL** set. If the answer is incorrect, the correct answer is displayed; press any key to resume play.
4. Follow your teacher’s instructions for how long to play the activity.
5. What level did you play? \_\_\_\_\_
6. What was your final score? \_\_\_\_\_
7. How many incorrect answers did you have? \_\_\_\_\_  
(Shown in top right corner of the screen.)
8. Write a paragraph describing the activity. Describe your strategy for playing.

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### Activities (continued)



**Scoring:** You get two attempts to solve each problem. You earn 2 points for a correct answer on the first try, 1 point for a correct answer on the second try.

You can earn up to 16 points.

### What Is My Property?

1. Look at the equation and decide which single property, out of these six, it represents.
  - Commutative +
  - Commutative \*
  - Associative +
  - Associative \*
  - Distributive \* Over +
  - Distributive \* Over -
2. Scroll through the property choices with  $\downarrow$  and/or  $\uparrow$ . To select a property, press **[ENTER]**. If the answer is incorrect, the correct answer is displayed; press any key to resume play. As you play the activity, record each equation and its property.

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3. What was your score? \_\_\_\_\_

 **Try-It!™ on Your TI-83 Plus or TI-73**

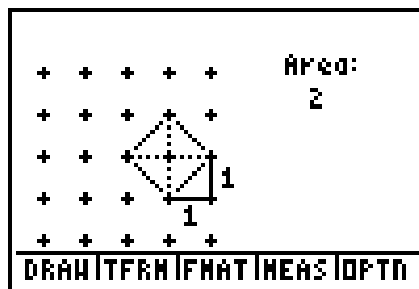
Investigate how the graphing calculator deals with irrational numbers.

In the **Overview**, you used the Pythagorean theorem to find the length of  $\sqrt{2}$ . Draw a right triangle so that the hypotenuse has length  $\sqrt{2}$ . Pythagoras (569–475 B.C.), a great Greek mathematician, discovered irrational numbers (numbers that are not rational and therefore are not ratios). There is a proof that, for example,  $\sqrt{2}$  cannot be written as a fraction.

**Hint:** Draw each leg with length 1 inch. The hypotenuse is  $\sqrt{1^2+1^2} = \sqrt{1+1} = \sqrt{2}$

Remember that  $\sqrt{2} \times \sqrt{2} = 2$ . Picture this by envisioning a square whose sides measure  $\sqrt{2}$  units. You created this length in your picture above.


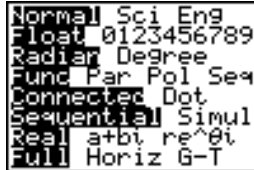
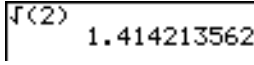
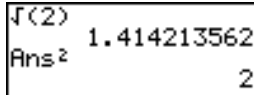
Look at the square whose side has a length of  $\sqrt{2}$  on the Geoboard screen below. Can you see that the area is  $\sqrt{2} \times \sqrt{2} = 2$  square units? Count it up! Shade in the area on the screen shown.



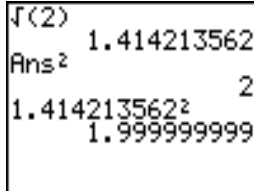
From the TI-73 Geoboard application

 **Try-It!™ on Your TI-83 Plus or TI-73 (continued)**

Find the graphing calculator decimal approximation for  $\sqrt{2}$ .

To Do This	Press	Display (TI-83 Plus shown)
1. Exit the Topics in Algebra 1 application and clear the Home screen.	$\boxed{2\text{nd}} \boxed{[\text{QUIT}]}$ $\boxed{\langle \text{EXIT} \rangle} \boxed{[\text{CLEAR}]}$	
2. Select the <b>Float</b> setting from the mode screen.  <b>Notes:</b> See  Tip™ 2: <i>Adjusting Your Graphing Calculator Settings</i> for details.  The TI-83 Plus mode screen varies slightly from the TI-73.	$\boxed{[\text{MODE}]}$ $\boxed{\downarrow}$ until <b>Float</b> is highlighted $\boxed{[\text{ENTER}]}$	
3. Return to the Home screen.	$\boxed{2\text{nd}} \boxed{[\text{QUIT}]}$	
4. Calculate the decimal approximation for $\sqrt{2}$ .	$\boxed{2\text{nd}} \boxed{[\sqrt{\quad}]} \boxed{2} \boxed{[)]}$ $\boxed{[\text{ENTER}]}$	
5. Square your result.  <b>Note:</b> <b>Ans</b> = previous answer. The graphing calculator remembers that you entered $\sqrt{2}$ .	$\boxed{[x^2]}$ $\boxed{[\text{ENTER}]}$	

Is  $\sqrt{2}$  equal to the decimal 1.414213562? It looks like the graphing calculator says this is true. Calculate  $1.414213562^2$  to see.

6. Calculate the square of 1.414213562.	$1 \boxed{.} \boxed{4} \boxed{1} \boxed{4} \boxed{2} \boxed{1} \boxed{3} \boxed{5} \boxed{6} \boxed{2} \boxed{[x^2]}$ $\boxed{[\text{ENTER}]}$	
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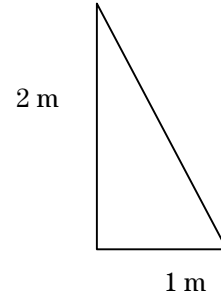
The answer shows 1.999999999, but you know you should get the answer 2.

Be careful! When you use your graphing calculator, you have to know your math. The graphing calculator can do amazing math, **but** it only shows you an approximation for many answers. You have to understand the problem before you use the graphing calculator. It is up to **you** to determine if graphing calculator answers are reasonable and how you will use them.

 **Try-It!™ on Your TI-83 Plus or TI-73 (continued)**

**Word Problem: Missy's Garden**

Missy wants to build a small fence around a garden in her backyard. The garden is in the shape of a right triangle. One leg is 2 meters and the other leg is 1 meter. The store sells fencing in tenths of a meter.



1. What is the exact perimeter of Missy's garden? Show all your work.

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**Math Hint:** Use the Pythagorean theorem to find the length of the third side of the garden, and then find the perimeter of the garden. (See the **Real Numbers Overview** on the graphing calculator.)

2. What length of fencing should Missy buy if the store only sells the fencing in tenths of a meter? Use the graphing calculator. Show all your work.

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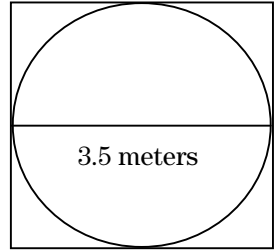
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**Graphing Calculator Hint:** After you calculate the approximate answer on your graphing calculator, try setting **MODE** so that **Float=1**. See  **Tip™ 2: Adjusting Your Graphing Calculator Settings** for details. This will give you one decimal place or tenths. Observe how the graphing calculator displays the results!

 **Try-It!™ on Your TI-83 Plus or TI-73 (continued)**

**Word Problem: Jose and Maria's Backyard Pool**

Jose and Maria have a circular pool in their backyard. Their parents would like to make a cover for the pool. They bought a square piece of material whose sides are the same length as the diameter of the pool. The diameter of the pool is 3.5 meters.



1. How much material will they have left over? Find the exact and approximate answers. Use the graphing calculator. Show all your work. (See Hints below.)

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

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2. Exact answer: \_\_\_\_\_
3. Approximate answer (to 3 decimal places): \_\_\_\_\_

**Graphing Calculator Hints:**

- Use **Float = 3** on your graphing calculator to display 3 decimal places.
- Press   to find the graphing calculator's approximation for  $\pi$ .

**Math Hints:**

- Area of a square:  $A = s^2$ , where  $s$  is the length of the sides of the square.
- Area of a circle:  $A = \pi r^2$ , where  $r$  is the radius of the circle (diameter =  $2r$ ).

**Number Sense: Real Numbers****Teacher Notes****Objectives**

- To illustrate the real number system in a Venn diagram.
- To identify real numbers as rational numbers  $\cup$  irrational numbers.
- To review writing rational numbers as terminating or repeating decimals.
- To review writing irrational numbers as nonterminating, nonrepeating decimals.
- To show physical representations of the irrational numbers,  $\sqrt{2}$  and  $\pi$ , and to review the Pythagorean theorem and the formula for finding the circumference of a circle.
- To state the real number system properties—commutative, associative, and distributive—as well as the identity and inverse properties.

**Math Highlights**

This section starts with the building of the Venn diagram of the real number system. Definitions of rational and irrational numbers are given. Two examples of irrational numbers,  $\sqrt{2}$  and  $\pi$ , are developed.  $\sqrt{2}$  is shown as the length of the hypotenuse of a right triangle with legs of 1 unit.  $\pi$  is shown as the circumference divided by the diameter for any circle. The statements of the properties of the real numbers follow.

**Common Student Errors**

- Many students may not have developed a solid understanding of number sets. Remind them that using rational and irrational numbers, they can name every location on a number line. Later in their studies of mathematics, this will be referred to as the Completeness Property of Real Numbers, which was an important discovery in mathematics. Later, they will also extend the real numbers to the complex numbers,  $a + b\sqrt{-1} = a + bi$ , which are numbers used, for example, in the study of the relationship between electricity and magnetism.
- Students probably have used  $22/7$  or  $3.14$  as an approximation of  $\pi$ . They may think that these values are exactly  $\pi$ , but they are not equal to  $\pi$ . This provides an opportunity to talk about approximations to several decimal places in real problems. The worksheet problems give students an opportunity to find exact and approximate answers. There are wonderful web sites that show  $\pi$  to millions of places. Mathematicians are still searching for more place values. This study of  $\pi$  requires the use of computers to assist the search.
- Some students may not be aware that the ratio of the circumference of a circle  $C$  divided by the diameter  $d$  is  $\pi$ .  $C/d = \pi$ . This may be confusing because they have been told that  $\pi$  is irrational and is **not** a ratio. Yet  $\pi$  came from a ratio of circumference to diameter. It turns out that either  $C$  or  $d$  is also irrational. The mathematics to prove this is not given at this level; therefore, students have to accept this without much explanation. This is a deep discussion that will not be of interest to some students, but other students may find it fascinating.



## Student Worksheet Notes with Answers

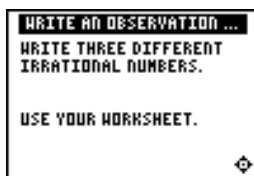
### Overview

Tell students:

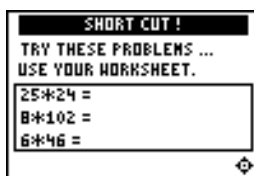
1. How to find the **Overview**, or tell them to review the instructions on the worksheet.
2. How to navigate the application, if they are not yet familiar with the application.
3. To scroll through the **Overview** on the graphing calculator. Point out new terms, definitions, and concepts, and tell students to look for them as they go through the **Overview**.

### Observations

The **Observations** help students understand number sense concepts relating to real numbers. Tell students how to find the **Observations**.



Students see this screen with three possible answers.



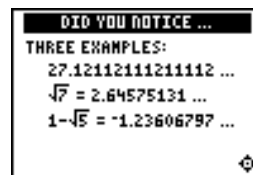
Students should show that they know how to use the properties of the real number computations as shortcuts without the graphing calculator.

Students see the answers on the next screen.

#### Observation 1

Write three different irrational numbers. Students show their work.

*Answers will vary.*

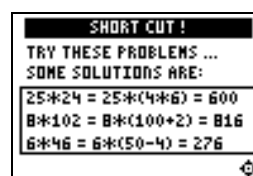


#### Observation 2

Try these problems.

Students use the real number properties to solve the problems, showing their work.

- *associative \* property:*  
 $25 * 24 = 25 * (4 * 6) = (25 * 4) * 6 = 100 * 6 = 600$
- *distributive \* over + property:*  
 $8 * 102 = 8 * (100 + 2) = (8 * 100) + (8 * 2) = 800 + 16 = 816$
- *distributive \* over - property:*  
 $6 * 46 = 6 * (50 - 4) = (6 * 50) - (6 * 4) = 300 - 24 = 276$



## Activities



**Scoring:** Every correct placement earns 2 points. When students give an incorrect answer, the correct answer displays.

The game automatically ends when they have answered incorrectly four times (shown in the top right corner), or they press **QUIT** to stop.

### Raining Reals

Tell students to:

1. Highlight a level (silver = less difficult; gold = more difficult), and press **ENTER** to select it.
2. Determine if the “raining” number is rational or irrational.
3. Press **←** to move the number into the **RATIONAL** set, or press **→** to move the number into the **IRRATIONAL** set. If the answer is incorrect, the correct answer is displayed; press any key to resume play.
4. Follow your instructions. For example, students can play:
  - until they have answered incorrectly four times (no time limit)
  - until a certain amount of time has expired (high score wins)
  - until a certain score has been reached (first student to reach the score with the fewest misses wins)
  - over a period of time (days, weeks, etc.) for tracking improvement of high scores
5. Record the level they played.
6. Record their final scores.
7. Record how many incorrect answers they had.

(Shown in top right corner of the screen.)

8. Write their strategy for playing the game.



**Scoring:** Students get two attempts to answer. They earn 2 points for a correct answer, 1 point for a correct answer on the second try.

They can earn up to 16 points.

### What Is My Property?

Tell students to:

1. Look at the equation and decide which one property, out of these six, it represents:
  - Commutative +
  - Commutative \*
  - Associative +
  - Associative \*
  - Distributive \* Over +
  - Distributive \* Over -
2. Scroll through the choices with **↓** and/or **↑** and press **ENTER** to select the correct property. If the answer is incorrect, the correct answer is displayed; press any key to resume play. As they play the activity, record each equation and its property.
3. Record their scores.

### Try-It!™ on Your TI-73 or TI-83 Plus

Tell students to:

- Use the Pythagorean theorem to draw a triangle whose two legs = 1 unit and whose hypotenuse =  $\sqrt{2}$  units.
- Shade in the area on the Geoboard screen on the worksheets.
- Go through the keystroke example to:
  - Find the graphing calculator decimal approximation for  $\sqrt{2}$ .
  - Understand how the graphing calculator approximates numbers.
  - Understand that they must be conscientious about the mathematics involved.

### Word Problem: Missy's Garden

Remind students to:

- Use the Pythagorean theorem to find the third side of the garden and then the perimeter.
- Set the decimal mode notation (**MODE**) to **Float** and then to **1** (answer rounded to tenths), so they can see how the graphing calculator displays answers.

1. *Exact perimeter:  $1 + 2 + \sqrt{5}$  meters*

2. *Length of fencing rounded to tenths: 5.2 meters*

**Note:** The number rounds down. This is mathematically correct, but impractical in the real world, where Missy would need to purchase 5.3 meters in order to fence the garden. You may want to discuss meaningful interpretation of word problems with the students.

### Word Problem: Jose and Maria's Backyard Pool

If necessary, review the formulas for area of a square and area of a circle. They are shown on the worksheet.

*Covering material left over:*

1. *Exact: Area of square - area of circle =  $(3.5)^2 - \pi(3.5/2)^2 = 12.25 - 3.0625\pi$  square meters*

2. *Approximate: 2.629 square meters (3 decimal places; use **Float=3**)*