Graphs of Sine, Cosine, and Tangent

ID: 8314

Time Required
45 minutes

Activity Overview

Students are introduced to the shapes of the sine, cosine, and tangent graphs. They will use a unit circle to calculate the x and y lengths of a triangle placed at different values along the circle. The tools provided will support students as they collect and organize data and build and test conjectures.

Topic: Trigonometric Functions

- *Right triangle trigonometry and the unit circle*
- *Shapes of the graphs of y = cos(x), y = sin(x), and y = tan(x)*

Teacher Preparation and Notes

- *Students should be introduced to the unit circle before this activity. They should be familiar with special angles and the values of (x, y) at these angles*
- *The first part of this activity is intended to be **teacher-led**. Students may be able to complete the end of the activity on their own, in small groups, or in pairs.*
- *Before beginning the activity, make sure all handhelds are in **Degree** Mode. To do this, go to the **Home** menu (on), and select **Settings > Settings > Graphs & Geometry**.*
- *Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.*
- ***To download the student TI-Nspire document (.tns file) and student worksheet, go to education.ti.com/exchange and enter "8314" in the keyword search box.***

Associated Materials

- *GraphsSinCos_Student.doc*
- *GraphsSinCos.tns*

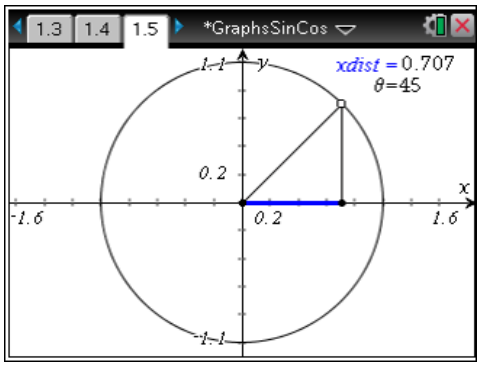
Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Round and Round She Goes… (TI-Nspire technology) — 12386*
- *The Unit Circle (TI-Nspire technology) — 9404*

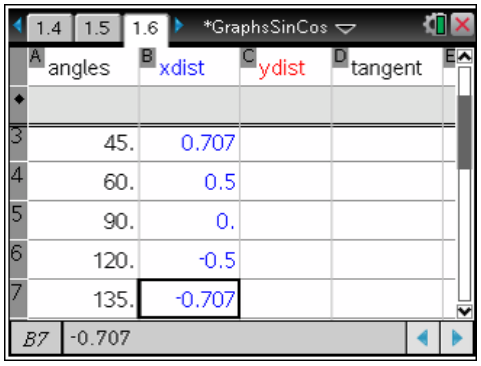
Investigating Relationship Between Dimensions

1. Students should go to page 1.5 and gather data. Students will move the triangle around the unit circle, using the point on the circle. At the special angles, or as close as possible, students should stop and put the values for the distance of the triangle along the *x*-axis (***x*dist**) in the table on the worksheet for the appropriate angle.



Students should then copy the information from the table into Column B on page 1.6.

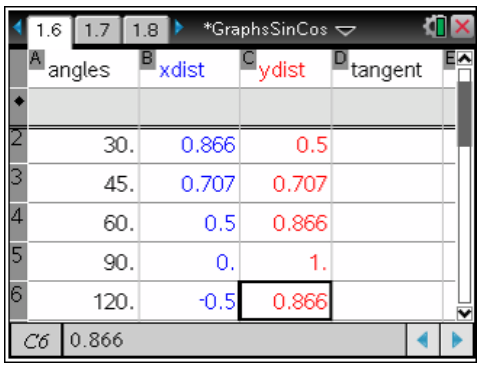
If you wish, students can collect more points than those at special angles—this will make a more complete graph once the data is plotted.



2. Now have students repeat the process with the *y*-values on page 1.8 and answer some questions about the shape of the graph.

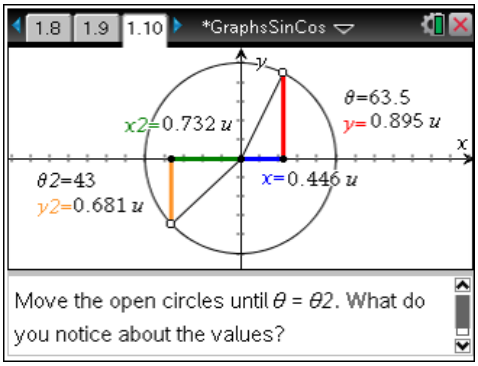
Students should understand that when the "distance" along the *x*-axis or *y*-axis is negative, the *x*dist or *y*dist values will need to be negative.

Students may notice that the magnitudes of some of the values are the same, such as 0.5 and –0.5 at 60° and 120°. They may also notice the pattern the *x*-values and *y*-values follow between –1 and 1.



3. To get the values in Quadrants III and IV, students can look at page 1.10 and discuss how to use the data already collected to obtain the new data.

Students should move the two points on the circle and set the two angles equal to each other, where θ is in Quadrant I and $\theta 2$ is in Quadrant III.



TI-Nspire Navigator Opportunity: *Screen Capture*

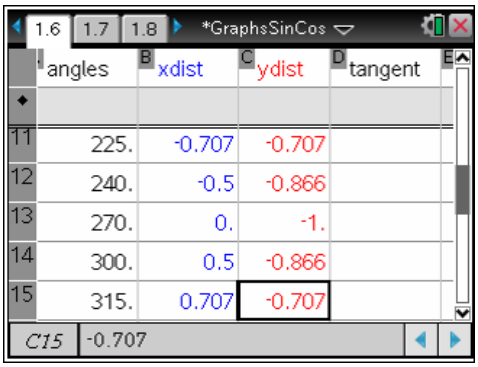
See Note 1 at the end of this lesson.

a. What do you notice about the two triangles? Have students compare the distances of the legs of the triangles. Students should see that the legs of the triangles are equal in length when the angles are the same. The only difference will be whether the values are positive or negative, which will depend on the quadrant.

b. How can you use the information from the first two quadrants to complete all the values on the table? Students should see they can match values in Quadrants III and IV to values for the same reference angle in Quadrants I and II. They need to determine the sign of the result based on the quadrant.

Students should also see that they can add 180° to the angles they already have to get the values in Quadrants III and IV.

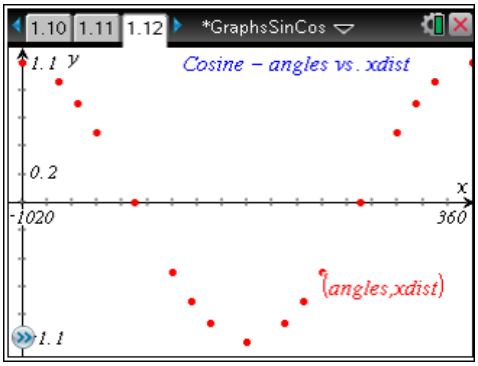
4. Have students complete the table of data for 180° to 360°. The data should be similar to those shown here.



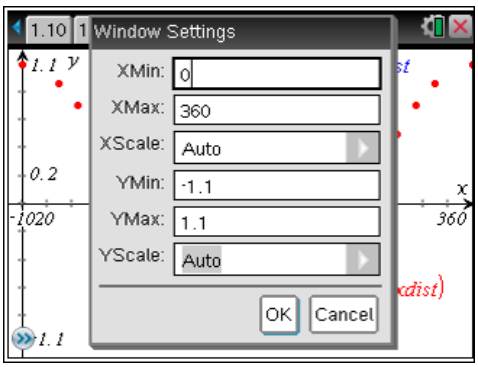
Graphing

5. Students will first graph $y = \cos(x)$ as a scatter plot on page 1.12 using the data they've collected (*angles*, *xdist*).

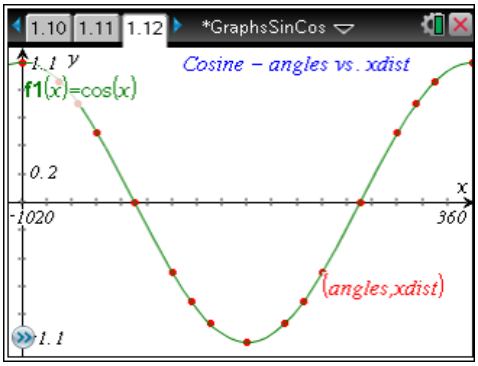
(**MENU > Graph Type > Scatter Plot**)



a. The window needs to be [0, 360] for the domain and [–1, 1] for the range. Students may have to adjust the window to see all points. You can have a class discussion about where these values come from and what they would be if you were graphing in radians.

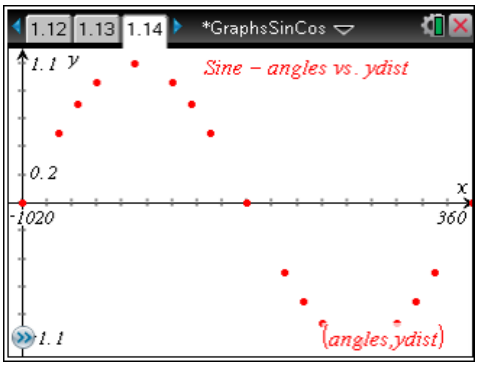


b. Students should then graph $y = \cos(x)$ in **f1**(x) and see that the graph goes through all the points.

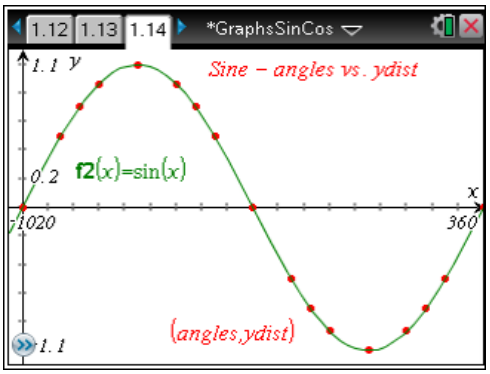


6. Now you will graph the sine function on page 1.14.

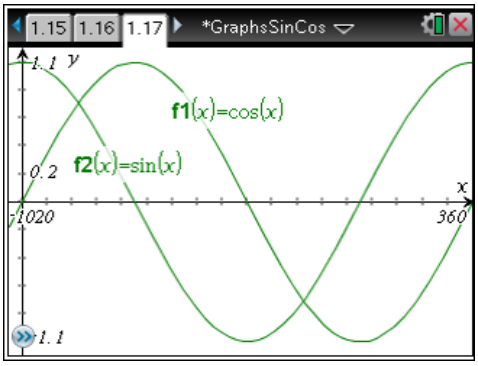
a. Students should create a scatter plot in **s2** on page 1.14 using the angles for x and $y$dist values for y, (*angles*, *ydist*).



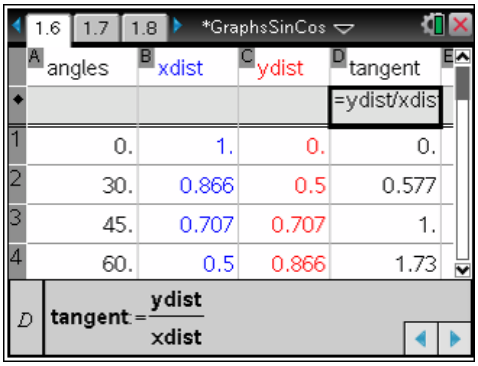
b. Students should graph $y = \sin(x)$ in **f2**(x) and see how the curve goes through all the points. Discuss the difference in the shapes with the students. Students should notice that the graphs for sine and cosine are the same, except they start at different points.



7. To get a better visual of the relationship between the two functions, students should sketch the graphs of $y = \cos(x)$ and $y = \sin(x)$ on the same graph as accurately as possible.

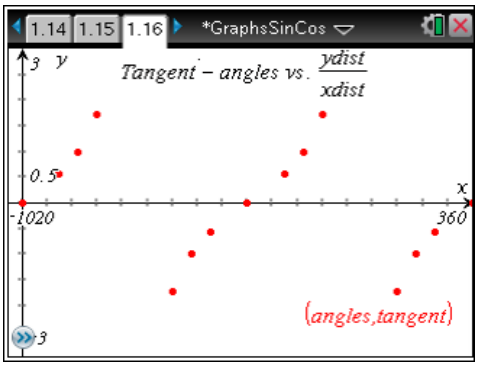


8. For tangent, the data and graph should look like the ones shown. Students can use a formula (**=*ydist/xdist***) to fill in the column for the tangent values.

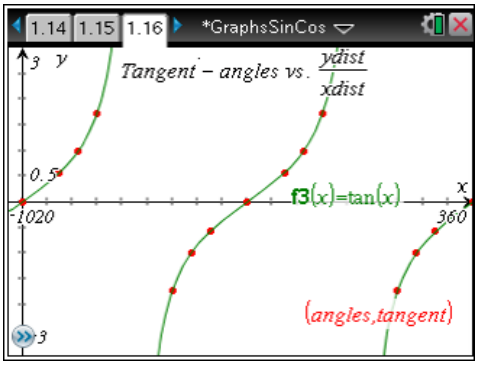


Students should repeat the same steps with the tangent to create a scatter plot and then graph the tangent function on page 1.16.

Students should see that the tangent graph is zero when sin(*x*) is zero, and undefined when cos(*x*) is zero. Make sure that students understand tangent is undefined at those points because you can't divide by zero.

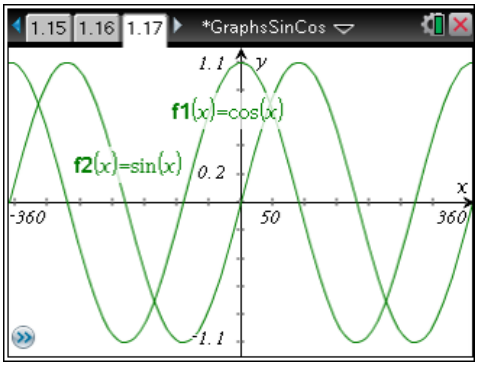


You can also discuss what happens on the graph when the function is undefined and explain asymptotes.



Extension

After students have looked at the graphs, you can have them think about and discuss what would happen if you extended the window or went to negative or positive infinity. You can also discuss the period of the functions.



Student Solutions

angles (in degrees)	*x*dist (cos(*x*))	*y*dist (sin(*x*))	tangent (sin(*x*)/cos(*x*))
0.	**1**	**0**	**0**
30.	**0.866**	**0.5**	**0.577**
45.	**0.707**	**0.707**	**1**
60.	**0.5**	**0.866**	**1.73**
90.	**0**	**1**	**undefined**
120.	**–0.5**	**0.866**	**–1.73**
135.	**–0.707**	**0.707**	**–1**
150.	**–0.866**	**0.5**	**–0.577**
180.	**–1**	**0**	**0**
210.	**–0.866**	**–0.5**	**0.577**
225.	**–0.707**	**–0.707**	**1**
240.	**–0.5**	**–0.866**	**1.73**
270.	**0**	**–1**	**undefined**
300.	**0.5**	**–0.866**	**–1.73**
315.	**0.707**	**–0.707**	**–1**
330.	**0.866**	**–0.5**	**–0.577**
360.	**1**	**0**	**0**

TI-Nspire Navigator Opportunities

Note 1
Problem 1, *Screen Capture*
This would be a good place to do a screen capture to verify students are finding angles in the appropriate quadrants.