Introduction to the Fundamental Theorem

FundamentalTheorem.tns

Name _____

Class _____

Problem 1 – Constant Integrand

On page 1.3, you can drag the point labeled *x* along the *x*-axis. As you do, observe how the integral and its value are updated. Notice that the value of the integral represents the area of the shaded rectangular region.

1. Complete the table of input/output pairs below.

x	$\int_0^x 1.5dt$
1	
2	
3	
4	
5	

2. If $x = 0$, what is $\int_0^x 1.5dt$? Why?

3. For every 1 unit that *x* changes, how much does $\int_0^x 1.5dt$ change?

4. If you were to graph the ordered pairs (x, $\int_0^x 1.5dt$), what would the graph look like?

On page 1.5, you'll be able to graph the ordered pairs from your table. On the left side, simply copy your answers from question 1 into column B of the spreadsheet. As you do this, each point will be graphed on the right.

5. What does the graph look like? Was the graph what you predicted in question number 4?

6. If you changed the integrand from 1.5 to 0.5, what would the graph of (x, $\int_0^x 0.5dt$) look like?

Problem 2 – Non-Constant Integrand

On page 2.2, drag the point labeled *x* along the *x*-axis to change the upper limit of integration. In this case, the value of the integral represents the area of the shaded triangular region. You'll have to drag *x* to the left to get the first few values for the table below. You might try to calculate the outputs using Geometry before you get the values from the handheld.

7. Complete the table of input/output pairs below.

x	$\int_0^x \frac{t}{2} dt$
1	
2	
3	
4	
5	

8. If $x = 0$, what is $\int_0^x \frac{t}{2} dt$? Why?

9. Explain why, when *x* increases by 1, the value of $\int_0^x \frac{t}{2} dt$ does not increase by the same amount every time.

10. Is the graph of $\left(x, \int_0^x \frac{t}{2} dt \right)$ linear? Explain.

Problem 3 – An Integrand that Changes Sign

Drag the point *x* along the *x*-axis. Observe the calculated value of $\int_0^x \frac{t^2 - 13t + 22}{9} dt$.

11. At what value of *x* does the integral's value begin to decrease?

12. a. What are all values of *x* for which the integral's value is decreasing?

b. What is true of the integrand at these values of *x*?

13. a. What are all values of *x* for which the integral's value is increasing?

b. What is true of the integrand at these values of *x*?

14. a. What is the smallest value of the integral, and at what value of *x* is this value reached?

b. What happens with the integrand at this value of *x*?

15. Is the connection between the location of the minimum value of $\int_0^x \frac{t^2 - 13t + 22}{9} dt$ and the sign change of the integrand from negative to positive one that you have seen before? If so, in what context?