# *NUMB3RS* Activity: Seeing Everyone
# Episode: “Under Pressure”

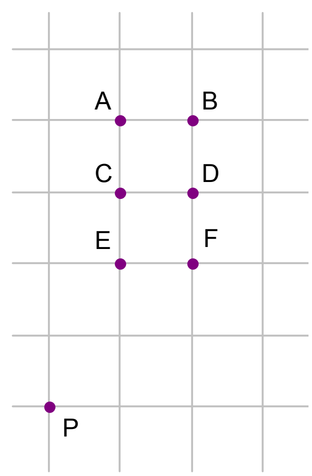
**Topic:** Visible lattice points in the coordinate plane **Grade Level:** 9 - 10
**Objective:** Find lines of sight among lattice points in a grid.
**Time:** 20 - 30 minutes
**Materials:** TI-83 Plus/TI-84 Plus graphing calculator (optional)

## Introduction

In “Under Pressure,” Charlie is aiding in the search for the person (or people) in charge of a terrorist cell composed of several independent, small groups of people who do not communicate with each other. Everyone involved is intentionally given a very limited perspective. As a metaphor, Charlie suggests picturing a bunch of kids playing in a pool and an adult in the pool watching them. Even though the adult is close to the kids, he or she cannot see everything that is happening. The most advantageous position for a lifeguard is outside the pool on an elevated chair. He concludes that, similarly, the leaders they are trying to find have the “widest view of the network.” In this activity, the idea of finding an advantageous position of someone with a “wide view of the network” is explored using the analogous problem of finding a points on a lattice from which several other sets of points are visible.

## Discuss with Students

A *lattice point* is a point with coordinates that are integers. Imagine the members of the small groups in the terrorist cell as lattice points and the leaders as observers located at other lattice points. Lines of sight are segments joining lattice points. A lattice point Q in a set of lattice points S is *visible* from an external point P if segment PQ does not contain any lattice points in S other than Q. If, for example, S = {A, B, C, D, E, F} and the observer is at P, then A is visible from P, but B is not (because segment PB contains point E). Determining whether one point is visible from another lattice point requires analyzing the value of the slope of the segment joining the two points.




For the last three problems in this activity, imagine the members of the terrorist cell are lattice points, the leaders are other lattice points, and, in addition, there is a bystander at every other lattice point. A lattice point Q in a set of lattice points S is *clearly visible* from an external point P if segment PQ does not contain any lattice points other than P and Q. For example, if S = {A, B, C, D, E, F} and the observer is at P, then E is clearly visible from P, but F is not because segment PF contains another lattice point. Determining whether one point is clearly visible from another lattice point also requires analyzing the value of the slope of the segment joining the two points.

**Student Page Answers:**
**1.** *(4, 4), (5, 5),(6, 3), and (6, 4)* **2.** *(3, 3), (3, 4),(3, 5), (5, 3), (5, 5), (6, 4)* **3.** *There are many, but (6, 3) is one possibility.* **4.** *There are many, but (8, 0) is one possibility.* **5.** *a ≠ 1, a ≠ 2, b ≠ 1, b ≠ 2, a ≠ b, (a, b) ≠ (3 ,0), (a, b) ≠ (0, 3)* **6.** *from the origin: GCD(a, b) ≠ 1; from (3, 1): GCD(a – 3, b – 1) ≠ 1* **7.** *{(0, 0),(1, 0)} is one choice.* **8.** *Suppose the six points are externally visible from (a, b) where a is odd and b is even. Then a – 1 and b – 2 are both even and (1, 2) is not visible from (a, b). Similar arguments can be used for other parity choices of a and b.*


Tom Butts, UTDallas, Richardson, TX
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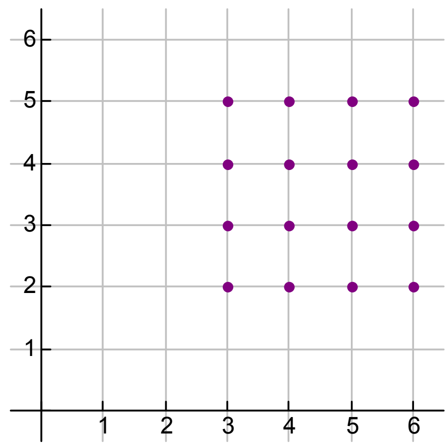
Name: ______________________________ Date: ____________________

# *NUMB3RS* Activity: Seeing Everyone

In "Under Pressure," Charlie is aiding in the search for the people in charge of a terrorist cell composed of several independent small groups who do not communicate with each other. Everyone involved is intentionally given a very limited perspective. As a metaphor, Charlie suggests picturing a bunch of kids playing in a pool and an adult in the pool watching them. Even though the adult is close to the kids, he or she cannot see everything that is happening. The most advantageous position is that of a lifeguard outside the pool on an elevated chair. Charlie concludes that, similarly, the leaders they are trying to find have the "widest view of the network." In this activity, the idea of finding an advantageous position of someone with a "wide view of the network" is explored using the analogous problem of finding a points on a lattice from which several other sets of points are visible.

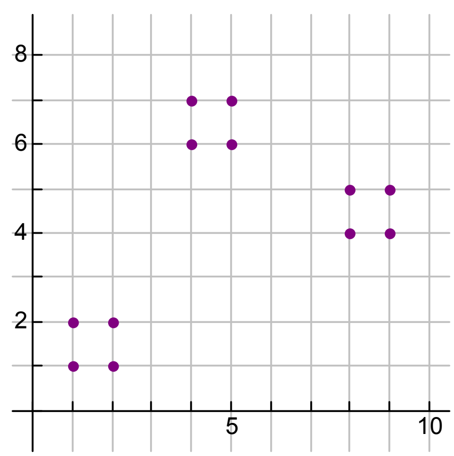
Imagine a member of the terrorist cell is standing at each of the 16 points with integer coordinates inside the square bounded by (3, 2), (6, 2), (6, 5), and (3, 5). A point with integer coordinates is known as a *lattice point*. Now imagine that you are the leader located at another lattice point outside this square. Which people can you see? A point Q in this set of 16 points is *visible* from an external point P if segment PQ does not contain any of the 16 points other than Q.



**1.** Suppose you are located at (0, 0). Which members in this 4 × 4 group with $3 \le x \le 6$ and $2 \le y \le 5$ are you **unable** to see?

**2.** Which lattice points in this 4 × 4 grid with $3 \le x \le 6$ and $2 \le y \le 5$ are **not** visible from the point (3, 1)?

**3.** To search for a possible leader, consider the three groups in the figure to the right. Find the point P from which each of the lattice points in the three groups is visible.





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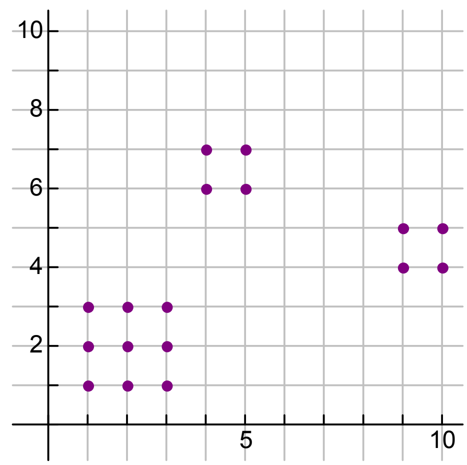
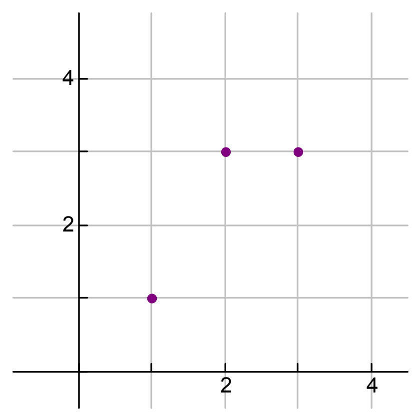
**4.** Consider the three groups shown in the figure to the right. Find the point P from which each of the lattice points in the three groups is visible.



**5.** Let S = {(1, 1), (1, 2), (2, 1), (2, 2)} and P = (*a*, *b*) be an external point in the first quadrant. Under what conditions on *a* and *b* will each point in S be visible from P?

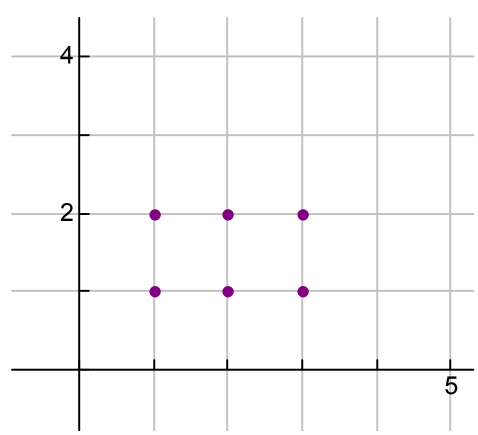
Now, imagine the leaders and members of the terrorist cell are standing at lattice points, and there is a bystander standing at each of the other remaining lattice points. A lattice point Q is *clearly visible* from another lattice point P if segment PQ does not contain any lattice points other than P and Q. Then, for example, (2, 3) is clearly visible from (1, 1), but (3, 3) is not because the segment joining (1, 1) and (3, 3) contains (2, 2).



**6.** If (*a*, *b*) is a lattice point in the first quadrant, under what condition on *a* and *b* will the point (*a, b*) **not** be clearly visible from the origin? From (3, 1)?

**7.** Find two observation points so that every point in the 2 × 3 array is clearly visible from at least one of these two points.

**8.** Explain why there is no single observation point from which every point in this 2 × 3 array is clearly visible.

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***The goal of this activity is to give your students a short and simple snapshot into a very extensive math topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.***

# Extensions

## For the Student

**1.** Let S be all points in the rectangle whose corners are (1, 1), ($r$, 1), ($r$, $s$), and (1, $s$). Prove that all points of S are visible from the point ($rs - s + r$, $s + 1$).

**2.** A rectangle has corners (1, 1), ($r$, 1), ($r$, $s$), and (1, $s$). Each point in the rectangle is clearly visible from at least one of two points. Find the largest possible values of $r$ and $s$.

**3.** What is the smallest value of $n$, for which it requires three points, so that every point in the square with vertices of (1, 1), (1, $n$), ($n$, 1), ($n$, $n$) is clearly visible from at least one of these three points?

**4.** In 3-space, under what condition will be point ($a$, $b$, $c$) **not** be clearly visible from the origin (0, 0, 0) if ($a$, $b$, $c$) is in the first octant?

## Reference

For more information about lattice points, read "Seeing Dots: Explorations in the Visibility of Lattice Points" by Josh Laison and Michelle Schick. To download this paper, go to:
**faculty1.coloradocollege.edu/~jlaison/seeing_dots.pdf**

## Additional Resources

- The topic of "visible lattice points" was first in the *NUMB3RS* activity "The Orchard Problem." To download this activity, go to **http://education.ti.com/exchange** and search for "7737."

- Examine the following Web sites and discover the connection between lattice points visible from the origin and Farey Sequences or Series:
**http://mathworld.wolfram.com/FareySequence.html**
**http://www.cut-the-knot.org/ctk/PickToFarey.shtml**


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