

# Parabolas – Paper Folding



## Student Activity

7 8 9 10 11 12



What do satellite dishes, car headlights, bouncing balls and waterspouts have in common? They are all related to the elegant shape and form of a parabolic curve. Satellite dishes and car headlights rely on the physical properties whilst bouncing balls and water fountains are acted upon by gravitational forces which help define their path. When it comes to parabolas, there are many instances of mistaken identity, curves that are look parabolic, but belong to a different family. Consider a chain supported at either end, is it parabolic? Can it be modelled by a parabola? What about the arch of the Sydney Harbor Bridge, is it an inverted parabola?



<https://youtu.be/7JMA6p8CNNs>

In this activity you will fold an A4 piece of paper repeatedly to create an envelope. The video link above provides a set of instructions for both the paper folding and digital creation.



## Step 1

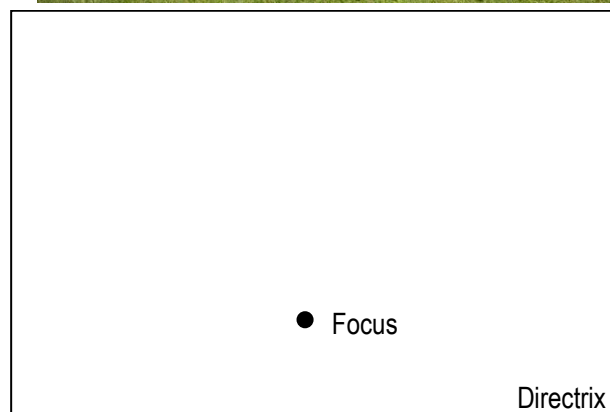
Use an A4 piece of paper in 'landscape' orientation.

Place a clearly visible dot horizontally centred somewhere between 2 to 8cm from the base of the page.

Label the dot: **Focus**

Label the bottom edge of the page: **Directrix**

*As shown in the diagram opposite.*

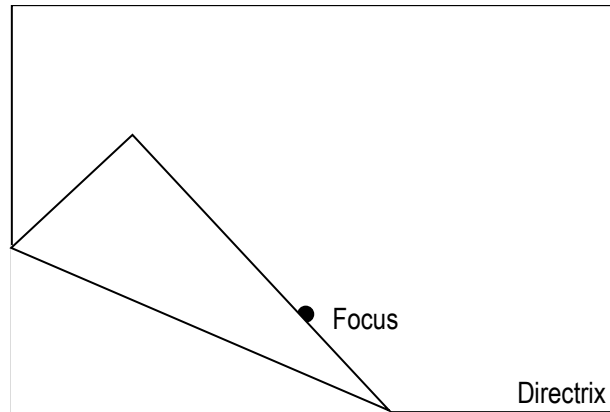


**Step 2**

Make a fold in the page so that the bottom edge of the paper (the directrix) just touches the dot (focus).

Press firmly on the fold so that it leaves a permanent crease in the paper.

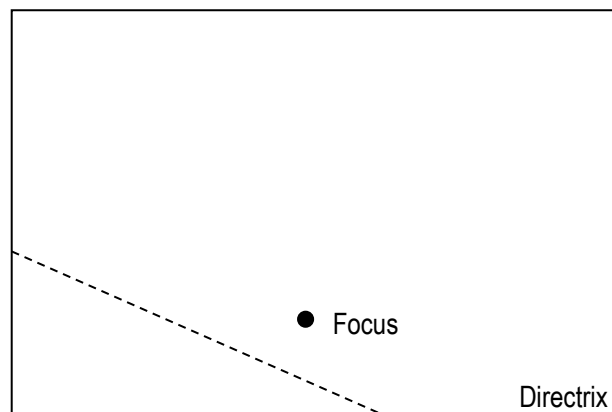
*As shown in the diagram opposite.*

**Step 3**

Unfold the paper to reveal the crease.

Use a pen or pencil and a ruler to trace over the crease.

*As shown in the diagram opposite.*

**Step 4**

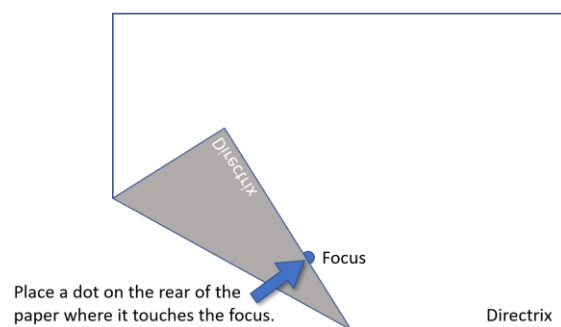
Repeat steps 2 and 3 until at least 20 **different** creases are formed on the page; approximately 10 on each side of the focus.

**Question: 1.**

The family of crease forms an 'envelope' or curve, describe the shape of the envelope.

Using a fresh piece of paper, repeat steps 1 to 3 to create a single fold. This time, when the paper is folded:

- Put a visible dot (P) on the bottom of the page (directrix) where it touches the focus (F).
- Unfold the paper and draw a line from the focus to this dot (PF).
- Measure the angle between the line (PF) and the fold.
- Measure the distances from P to the fold and from the focus (F) to the fold.

**Question: 2.**

What is the angle between the line (PF) and the fold?

**Question: 3.**

Compare the distances from the dot (P) to the fold (line) and also from the focus (F) to the fold, comment on your findings.

**Calculator Instructions: Digital Paper Folding**

Open a new TI-Nspire document and insert a Geometry application.

Press: **P**

Place a point in the middle of the page (horizontally) and slightly towards the base of the page.

**Note:**

P provides a short-cut to draw a point. The point tool can also be accessed through the menu by selecting points and lines.

Press:

**[menu] > Points & Lines > Line**

Start the line towards the bottom left of the page. Click to place the first point on the line, then, hold down the shift key and move the pen to the right. The shift key will make it easier to draw a horizontal line.

Press:

**[menu] > Construction > Perpendicular Bisector**

Click on the original point, followed by the horizontal line. A perpendicular bisector will be drawn between the original point and the newly created point on the horizontal line. Press **[esc]** to release the tool.

Grab the newly created point on the horizontal line and move it left and right, notice how the perpendicular bisector works.



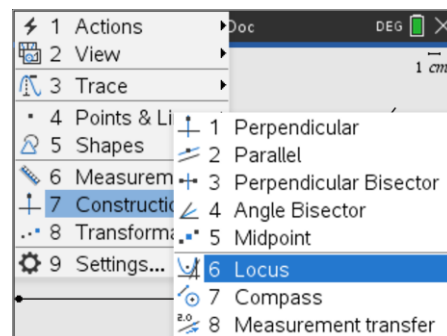
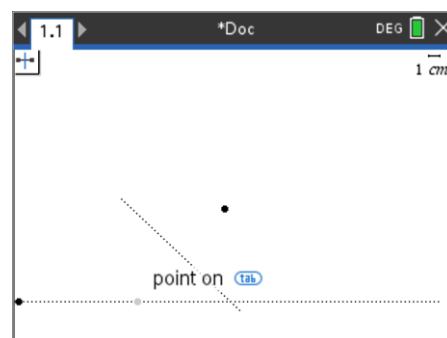
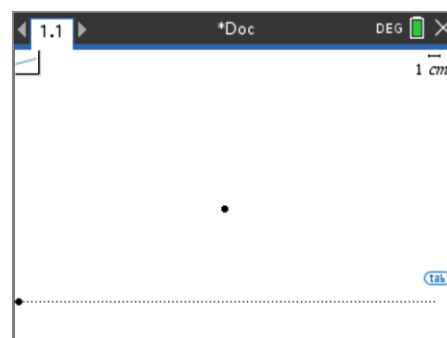
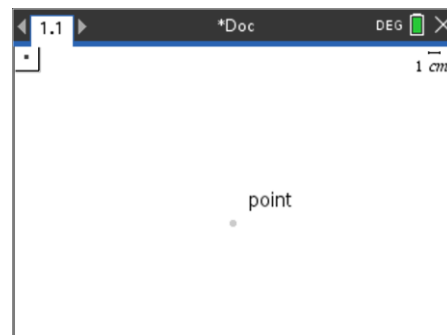
- The option to automatically label points is switched 'off' in the document shown, this is optional. It has been off here to keep the page uncluttered.
- The perpendicular bisector can be lengthened in either direction by grabbing the respective end of the line.

A locus is a curve or shape made by all the points satisfying a particular condition or set of conditions. In this situation, the perpendicular bisector is governed by the location of the line and point.

The locus can be constructed by:

**[menu] > Construction > Locus**

Once the locus tool is active, click on the original point followed by the perpendicular bisector.



The family of straight lines should resemble your paper folding outcome. The difference here is that the result is dynamic. Explore what happens when the original point is moved or the horizontal line is moved.

**Question: 4.**

The series of lines left by the locus forms an 'envelope'. What is the shape of the envelope?

**Question: 5.**

Explain how this digital image (locus) relates to the one completed using pencil and paper.

Insert a Graph application into your current document. Press **[esc]** to exit the equation entry line.

Press: **P**

Place the point on the Y axis at: (0, 2).

**Note:**

Placing the point over the hash mark on the y-axis will lock the point onto hash marks along y-axis.

Press:

**[menu] > Geometry > Construction > Perpendicular**

Place the perpendicular line on the point (0, -2). (Y-axis)

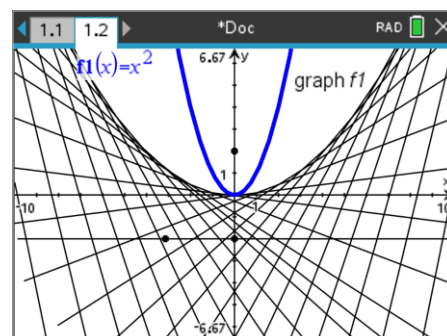
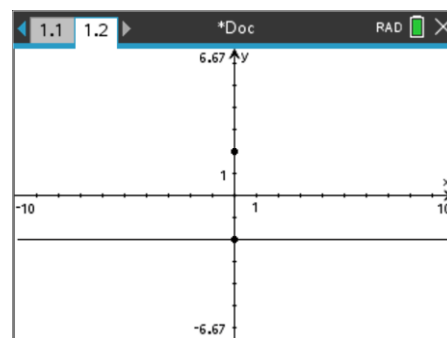
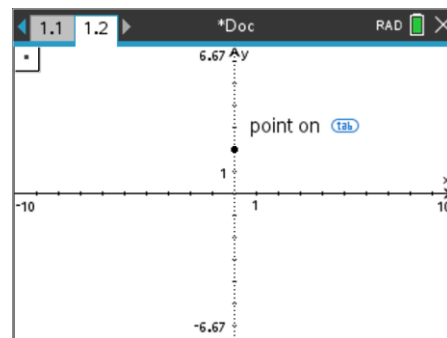
The point and line can be moved later, for now, these two locations will make things easier once the envelope has been created.

Repeat the instructions for creating a perpendicular bisector and locus.

Once the locus has been created (shown opposite), navigate to the equation entry line and enter the equation:

$$f_1(x) = x^2$$

The graph can be transformed by 'grabbing' either the vertex or the 'sides' of the parabola.



**Question: 6.**

Can you get the parabola to fit the curve?