Mr. Taylor, I Presume?

TIMATH.COM: CALCULUS

TEACHER NOTES

ID: 10257

Activity Overview

In this activity, students will use the TI-89 to explore Taylor polynomials graphically and analytically. They will also graphically determine the interval where the Taylor polynomial closely approximates the function it models.

Topic: Series and Taylor Polynomials

- Display in a spreadsheet the first few terms of a Taylor series approximation to $f(x)$ for a given value of x and compare the value of the Taylor approximation with the value of $f(x)$.
- Use Taylor Polynomial (in the Calculus menu) to verify the manual computation of a Taylor Series.
- Graph a function and its Taylor polynomials of various degrees to show their convergence to the function.

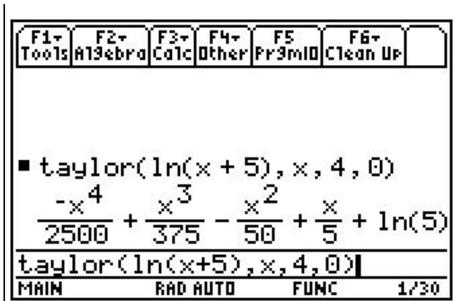
Teacher Preparation and Notes

- Students should be familiar with basic differentiation prior to the beginning of this activity.
- Students should be able to graph functions, and set up tables on their calculators. They should also be able to use commands and set up tables on their own.
- This activity is designed to be teacher-led.
- Before starting this activity, students should go to the Home screen and select **F6:Clean Up > 2:NewProb**, then press [ENTER]. This will clear any stored variables, turn off any functions and plots, and clear the drawing and home screens.
- **To download the student worksheet, go to education.ti.com/exchange and enter "10257" in the keyword search box.**

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- MacLaurin Polynomials (TI-89 Titanium) — 10128
- Exploring Taylor Series (TI-84 Plus family or TI-89 Titanium) — 5525
- Taylor Polynomials (TI-84 Plus family) — 4375



This activity includes screen captures taken from the TI-89 Titanium.

Compatible Devices:

- TI-89 Titanium

Associated Materials:

- MrTaylorIPresume_Student.pdf
- MrTaylorIPresume_Student.doc

Click HERE for Graphing Calculator Tutorials.

Introduction

This activity begins with students finding a polynomial given only the value of its derivatives at a specific *x*-value. This is at the heart of finding Taylor polynomials. If students have difficulty understanding that this method works, it may be necessary to reverse the process and have the students find the derivatives and observe that their values at the given *x*-value are the given values of the derivative.

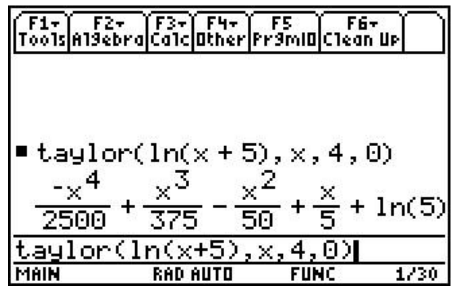
At the conclusion of the introductory problem, the Taylor polynomial form is found when centered at zero. At this time, place the general form of the Taylor polynomial when $x = 0$ on the board.

$$P_n(x) = \frac{f^{(0)}(a)}{0!}(x-a)^0 + \frac{f'(x)}{1!}(x-a)^1 + \frac{f''(x)}{2!}(x-a)^2 + ... + \frac{f^{(n)}(x)}{n!}(x-a)^n$$

Explain that when $a = 0$, this matches the form $P(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + ... + a_nx^n$.

Taylor Polynomials Centered at Zero

Students first work through an example to find a polynomial $P(x) = a_0 + a_1x + a_2x^2 + a_3x^3$ if $P(0) = 1$, $P'(0) = 1$, $P''(0) = 6$ and $P'''(0) = 9$.

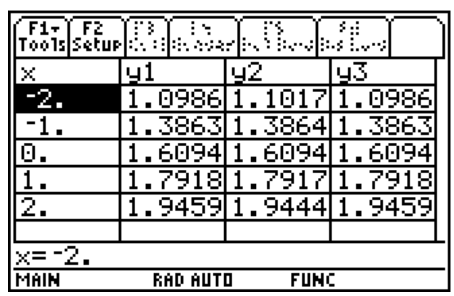


Next, students are asked to find the 4th degree Taylor polynomial that approximates $f(x) = \ln(x + 5)$ at $x = 0$. The derivatives of the function are:

$$f(0) = \ln(5) \approx 1.609$$

$$f'(x) = \frac{1}{x+5} \rightarrow f'(0) = \frac{1}{5}$$

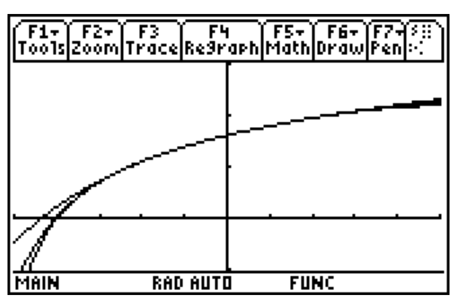
$$f''(x) = \frac{-1}{(x+5)^2} \rightarrow f''(0) = -\frac{1}{25} \qquad f'''(x) = \frac{2}{(x+5)^3} \rightarrow f'''(0) = \frac{2}{125} \qquad f^{(4)}(x) = \frac{-6}{(x+5)^4} \rightarrow f^{(4)}(0) = -\frac{6}{625}$$



x	y1	y2	y3
-2.	1.0986	1.1017	1.0986
-1.	1.3863	1.3864	1.3863
0.	1.6094	1.6094	1.6094
1.	1.7918	1.7917	1.7918
2.	1.9459	1.9444	1.9459

As the students compare the values of the Taylor polynomial with the values of the original function, they will notice that the values are closest at the center (the *x*-value where the derivatives were found) and become farther apart the further the *x*-values are from the center (the graph demonstrates this even further). Point out to students that not all Taylor polynomials have such a large interval as this one.

The table at the right has the 4th power and 10th power Taylor polynomials as well as the function.



Students will also have a chance to explore different degrees of a Taylor polynomial. It would be best if you choose three larger values ahead of time. Have the students use the **Taylor** command to find the respective polynomials and graph the function with each of the larger degree polynomials in turn.

If time permits, have a few students find the 10th degree Taylor polynomial and graph it on the same set of axes as the 4th degree Taylor and the original function. This will demonstrate that a Taylor polynomial will only approximate the function over a given interval, no matter how large the degree of the Taylor Polynomial. It is important for students to know that a Taylor polynomial centered at 0 is known as a Maclaurin polynomial.

Taylor Polynomials Not Centered at Zero

When students work the problem of where the Taylor polynomial is not centered at zero, they will observe that the polynomial follows the original function for a much smaller interval. Remind students that they need to use $(x - a)^n$ instead of x^n.

The derivatives for this problem are found below.

	Polynomial	**Value at x = 1**
$f(x)$	$\frac{1}{2-x}$	1
$f'(x)$	$-(2-x)^{-2}(-1) = \frac{1}{(2-x)^2}$	1
$f''(x)$	$-2(2-x)^{-3}(-1) = \frac{2}{(2-x)^3}$	2
$f'''(x)$	$2(-3)(2-x)^{-4}(-1) = \frac{6}{(2-x)^4}$	6
$f^{(4)}(x)$	$6(-4)(2-x)^{-5}(-1) = \frac{24}{(2-x)^5}$	24

The Taylor polynomial can be defined as:

$$P(x) = \frac{1}{0!}(x-1)^0 + \frac{1}{1!}(x-1)^1 + \frac{2}{2!}(x-1)^2 + \frac{6}{3!}(x-1)^3 + \frac{24}{4!}(x-1)^4$$

or

$$P(x) = 1 + (x-1) + (x-1)^2 + (x-1)^3 + (x-1)^4$$

Pick three values from 0 to 1 (for instance 0.8, 0.5, 0.2) and find the 4th degree Taylor polynomial using the **Taylor** command. Graph each Taylor polynomial and the original function on the same set of axes separately. Students should notice that the interval increases as the center gets closer to zero. See the graphs below.