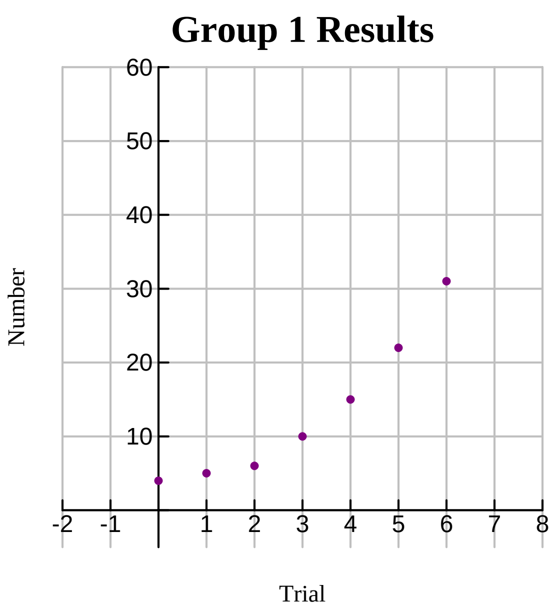
Exponential Growth and Decay - M&M's Activity

Activity 1 - Growth

1. The results from the experiment are as follows:

Trial	Group 1
0	4
1	5
2	6
3	10
4	15
5	22
6	31

2. The scatterplot of the result is as follows:



3. The exponential model for the data is:

Group 1 Results
Exponential Regression
regEQ(x) = 3.51849*1.43039^x
a = 3.51849
b = 1.43039
r = .992621

The value of *r* indicates that there is a strong positive correlation between the rule and the data.

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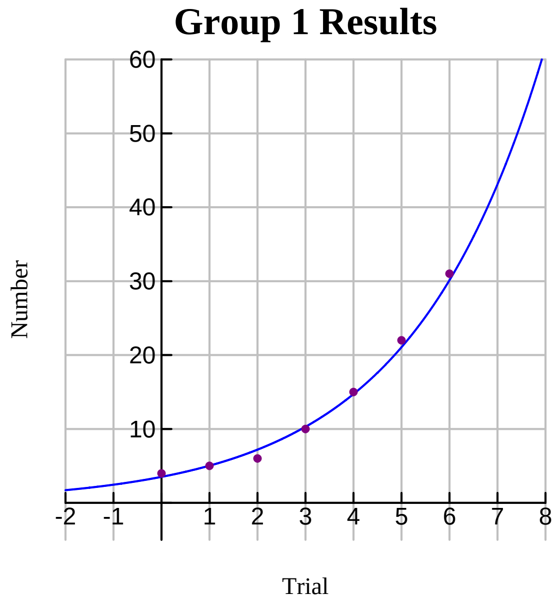
4. When graphed on the same set of axes as the data it can be seen that the curve is a good fit of the data.



The table below gives a comparison between the rule, correct to one decimal place, and the actual results. The percentage error column gives an indication of the closeness of the rule to the actual data collected.

The data point that appears to be most inaccurate is trial 2 where there is a 16.65% error. In this it seems that the error most likely occured in the trial itself - with fewer than the expected number of M&M's falling with the M upwards.

Trial	Group 1	Rule	% Error
0	4	3.5	-13.69
1	5	5.0	.65
2	6	7.2	16.65
3	10	10.3	2.89
4	15	14.7	-1.84
5	22	21.1	-4.42
6	31	30.1	-2.87
		Average	-.37

5. If we want to predict the number of M&M's on trial number 9 there are two ways we can do this:

a) Using the table feature

x	3.51849*(1.43039)^(x)
0	3.51849
1	5.03281
2	7.19889
3	10.2972
4	14.729
5	21.0683
6	30.1358
7	43.106
8	61.6584
9	88.1955
10	126.154
11	180.449
12	258.113

b) Substituting directly into the rule

$y(x) := 3.51849 \cdot 1.43039^{x}$

"Done"

$y(9)$

88.1955

This indicates that after 9 trials there should be 88 M&M's present.

6. To predict the number of trials to obtain 300 M&M's we can use the numerical solve feature

$\text{nsolve}\left(300 = 3.51849 \cdot 1.43039^{x}, x\right)$

12.4201

Therefore we would expect at least 13 trials before there would be 300 M&M's present.

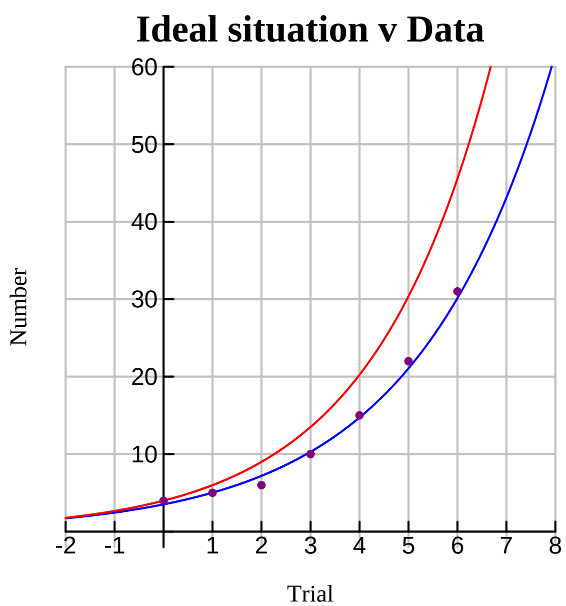
7. In the expression $N = a \cdot b^{x}$, the value of ***a*** represents the initial number of M&M's present.

That is, if $x = 0$, $N = a \cdot b^{0}$, $N = a$

The value of ***b*** represents the growth rate. In this case we expect the population to grow by half on every trial.

Hence in an "ideal situation" the rule should be $N = 4 \cdot (1.5)^{x}$

Graphing the ideal situation on the same axes as the rule generated and the original data shows that the data underestimates the expected result.



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8. Using the points $(0, 4)$ and $(4, 15)$ in an equation of the form $y = a \cdot b^x$ gives the following results:

$4 = a \cdot b^0$

$4 = a$

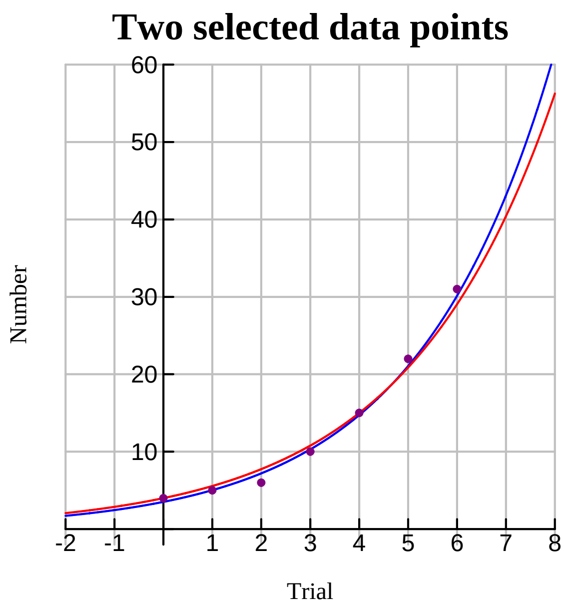
$15 = 4 \cdot b^4$

$b = \sqrt[4]{\left(\frac{15}{4}\right)}$

$b = 1.39$

Hence the rule is $y(x) = 4 \cdot (1.39158)^x$

Graphing this result on the same axes as the function proposed by the regression model indicates that the results are fairly accurate.



9. A spreadsheet comparing the three results is as follows:

x	Regression	Data points	Ideal
0	3.52	4.00	4.00
1	5.03	5.57	6.00
2	7.20	7.75	9.00
3	10.30	10.78	13.50
4	14.73	15.00	20.25
5	21.07	20.87	30.38
6	30.14	29.05	45.56
7	43.11	40.42	68.34
8	61.66	56.25	102.52

10.

a) If two M&M's were added on each trial then the rule that models the situation is $N = 4 \cdot (2)^x$

b) If three M&M's were added on each trial then the rule that models the situation is $N = 4 \cdot (2.5)^x$

c) In general, if *m* M&M's are added on each trial then the rule that models the situation is

$$N = 4\cdot\left(\frac{2+m}{2}\right)^x$$ or alternatively $$N = 4\cdot\left(1+\frac{m}{2}\right)^x$$

So the "perfect result" would be:

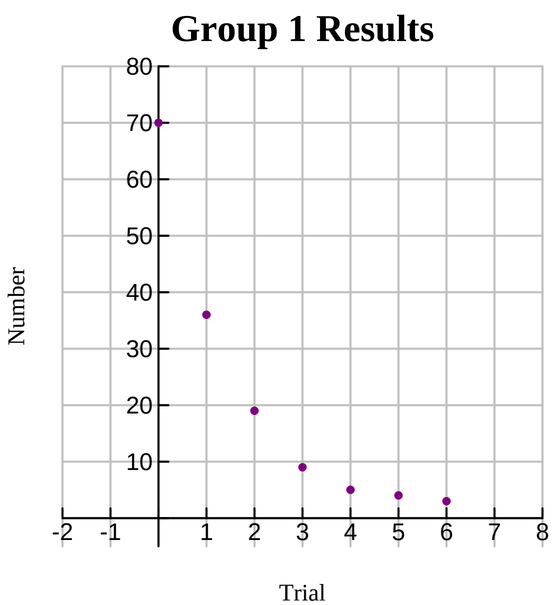
Trial	Adding One	Adding Two	Adding Three
0	4.0	4.0	4.0
1	6.0	8.0	10.0
2	9.0	16.0	25.0
3	13.5	32.0	62.5
4	20.3	64.0	156.3
5	30.4	128.0	390.6
6	45.6	256.0	976.6
7	68.3	512.0	2441.4

Activity 2 - Decay

1. The results from the experiment are as follows:

Trial	Group 1
0	70
1	36
2	19
3	9
4	5
5	4
6	3

2. The scatterplot of the result is as follows:



3. The exponential model for the data is:

Group 1 Results
Exponential Regression
regEQ(x) = 58.2428*.58152^x
a = 58.2428
b = .58152
r = -.984429

The value of *r* indicates that there is a strong positive correlation between the rule and the data.

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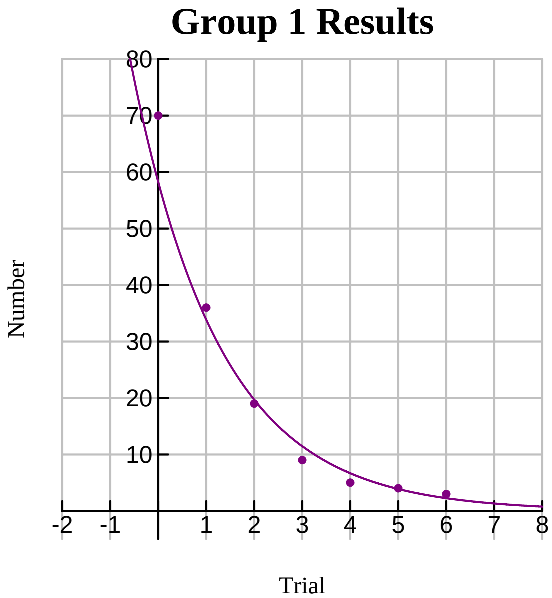
4. When graphed on the same set of axes as the data it can be seen that the curve is a reasonable fit of the data.



The table below gives a comparison between the rule, correct to one decimal place, and the actual results. The percentage error column gives an indication of the closeness of the rule to the actual data collected.

Trial	Group 1	Rule	% Error
0	70	58.2	-20.19
1	36	33.9	-6.29
2	23	19.7	-16.78
3	14	11.5	-22.23
4	5	6.7	24.93
5	4	3.9	-3.27
6	3	2.3	-33.20
		Average	-11.00

5. Assuming that you start with 900 M&M's, how many trials would you need before the experiment is over? Use a table to assist you.

Tracing backwards through the table until the total is approximately 900 gives a result of 5 trials. As the number is below one after 8 trials then the total number of trials needed is 13.

x	58.2428*(.58152)^(x)
-5	875.828
-4	509.312
-3	296.175
-2	172.232
-1	100.156
0	58.2428
1	33.8694
2	19.6957
3	11.4534
4	6.66041
5	3.87316
6	2.25232
7	1.30977

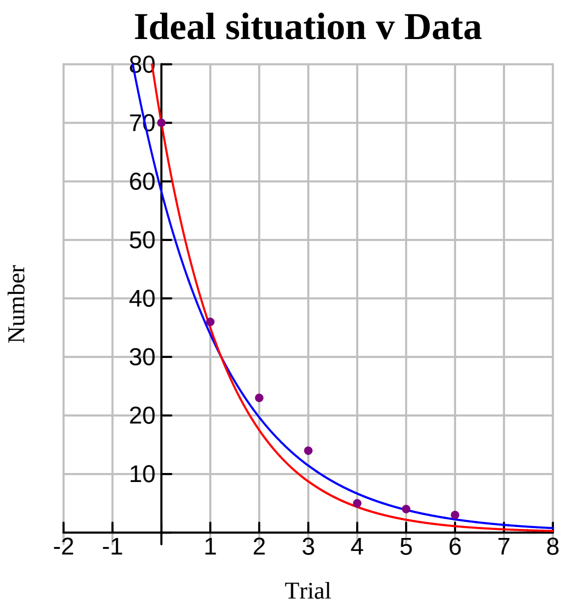
6. In the expression $N = a \cdot b^x$, the value of *a* represents the initial number of M&M's present.

That is, if $x = 0$, $N = a \cdot b^0$, $N = a$

The value of *b* represents the decay rate. In this case we expect the population to decrease by half on every trial.

Hence in an "ideal situation" the rule should be $N = 70 \cdot (0.5)^x$

Graphing the ideal situation on the same axes as the rule generated and the original data shows that the data underestimates the expected result.



7. A spreadsheet comparing the three results is as follows:

x	Regression	Ideal	
0	58.24	70.00	
1	33.87	35.00	
2	19.70	17.50	
3	11.45	8.75	
4	6.66	4.38	
5	3.87	2.19	
6	2.25	1.09	
7	1.31	.55	

8.

a) It is impossible to remove three M&M's for each M upward on each trial because the expectation is that there will be a halving of the population on each trial. Therefore if three M&M's are removed for each M upwards then 1.5 times the population is removed. Consequently the experiment would last only one trial.

b) The maximum number of M&M's that can be removed for each M upward on any one trial is one. This enables the population to halve each time.

c) In general, if *m* M&M's are added on each trial then the rule that models the situation is

$$N = 4 \cdot \left(\frac{2+m}{2}\right)^{x}$$

So the "perfect result" would be:

Trial	Adding One	Adding Two	Adding Three
0	4.0	4.0	4.0
1	6.0	8.0	10.0
2	9.0	16.0	25.0
3	13.5	32.0	62.5
4	20.3	64.0	156.3
5	30.4	128.0	390.6
6	45.6	256.0	976.6
7	68.3	512.0	2441.4