Hose Problem
INVESTIGATION USING TI –*n*spire CAS

The Hose Problem presented below can be used in a simplified version with Y10 students after they have learnt about quadratic functions. Full version can be used for extension students in Y10, as a transition activity from Y10 to Y11 and in Y11 Mathematical Methods and General Mathematics.

TI-nspire CAS was used to illustrate how its many features can be applied to solve the Hose Problem including Graphs and Tables, Calculator and its applications, Lists and Spreadsheets with regression lines and in general managing document and pages and using Computer Link software.

You may remember what it was like when you and your friends played under a hose, squirting each other and moving the hose around watching the paths traced out by the water jets (and thus by each water droplet in each jet). Imagine yourself with a hose now (or go out and experiment with a hose). Suppose you turn the tap on so that the water is coming out of the nozzle at a speed of 10 m/s. Now suppose you are lying on the ground holding the hose with your arm outstretched in front of you so that the water jet is horizontal (along the ground). Let this direction be that of the positive *x*-axis of a Cartesian coordinate frame, with your hand near the origin, the nozzle of the hose being at the origin.

1. Imagine (or go out and observe) the shape of the water jets as you rotate your hand, by fixed amounts, until the jet is vertical. With reference to the Cartesian axes shown below draw a sketch, based on your intuition or your observation, of the shape of a typical jet. What type of curve do you expect such a jet to trace out?
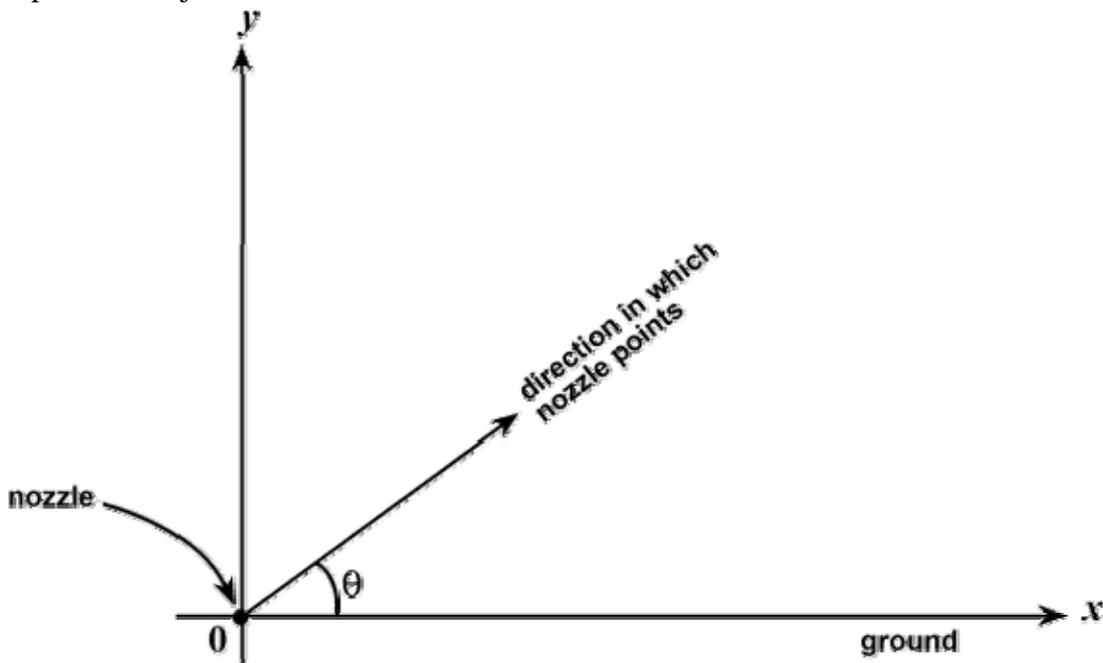
Suppose that the water jet leaves the nozzle at a given angle θ ($0° < θ < 90°$) to the horizontal, and assume that air resistance against the jet is negligible. Then it can be shown that, with reference to the axes above, the curve traced out by the jet has the equation

$$y = ux - 0.049(1+u^2)x^2, \text{ where } u = \tan\theta. \qquad \textbf{(equation 1)}$$

Let us call this 'equation 1'. The 0.049 is related to the initial speed of the water, 10 m/s, and acceleration due to gravity of 9.8 m/s^2.

Set up your new document in Degrees and Approximate by pressing ⌂, 8: System Info, 1: Document Settings. Those settings will only apply to the current document.
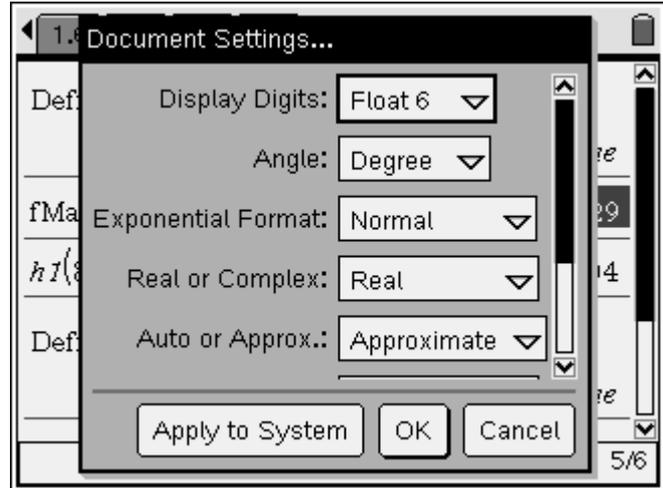
Select 1: Calculator. Define u and y. Check that it works for various values of θ.
Note: you need to have $u \times x$ entered, not ux, or the calculator will treat it as a single variable ux.

Symbols can be located by pressing ⌃ ⌨.
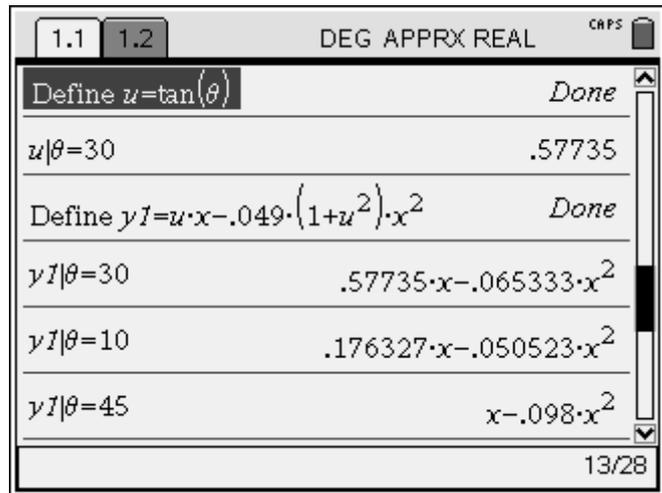
2. i. For the case where $\theta = 30°$, find the position of the highest point reached by the jet and the position of the point where the jet strikes the ground, and sketch the graph of the curve traced out by the jet over the appropriate domain.

Copy the equation for the angle 30 degrees by highlighting and ⌃ C. Open a new page ⌂, 2: Graphs, Paste your equation into one of the functions.

Set up your Window: ⒨ 4: Window, 1: Window Settings. Alternatively change the values on coordinate axes.

How can you tell where the *x*-intercepts are, given the equation in Y1? Class discussion.

Go back to page 1.1 and factor the equation to see the *x*-intercepts.

You may wish to restrict the domain if the function so that $y \ge 0$ by entering /x>0 and x<8.83 after the equation as seen in the screen above.

1.1	1.2	1.3	1.4 ▶ DEG APPRX REAL		CAPS

.57735026918963·x − .00533333333332·x

$.57735 \cdot x - .065333 \cdot x^2$

factor(.57735026918963·x − .0653333333333 ▶

$-.065333 \cdot x \cdot (x - 8.83699)$

y1|θ=45 $x - .098 \cdot x^2$

factor$(x - .098 \cdot x^2)$ $-.098 \cdot x \cdot (x - 10.2041)$

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Use (menu) 5: Trace to locate the coordinates of the turning point.

To hide the function entry line press (ctrl) G. And (ctrl) G again to see the entry line.

Use (tab) to switch between the entry line and the graph.

1.1	1.2	1.3	DEG APPRX REAL		CAPS

.86 *y*

f6(x)=.57735·x−.065333·x²|x>0 and x<8.83
θ=30°

$(4.40995, 1.27551)$

0.2

0.1   *x*

                                                9.25

ii. On the same set of axes, draw curves corresponding to three other angles θ, in each case locating carefully the highest point and the point where the jet strikes the ground.

Repeat the above for other angles e.g. 45, 60 and 75 degrees to have a good spread.

1.1	1.2	DEG APPRX REAL		CAPS

y1	θ=30	$.57735 \cdot x - .065333 \cdot x^2$
y1	θ=10	$.176327 \cdot x - .050523 \cdot x^2$
y1	θ=45	$x - .098 \cdot x^2$
y1	θ=60	$1.73205 \cdot x - .196 \cdot x^2$
y1	θ=50	$1.19175 \cdot x - .118594 \cdot x^2$
y1	θ=70	$2.74748 \cdot x - .418883 \cdot x^2$

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Restrict the domain again by looking at the factorised form.

$$f11(x) = x - .098 \cdot x^2 | x > 0 \text{ and } x < 10.204$$
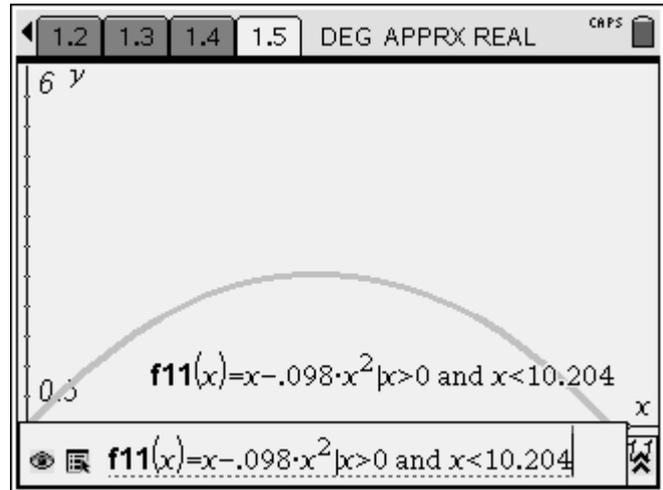
Write down the angle next to the graph instead the equation from (menu)1: Tools, 5: Text. Note that angle $\theta = 45^o$ is in $f11(x)$.
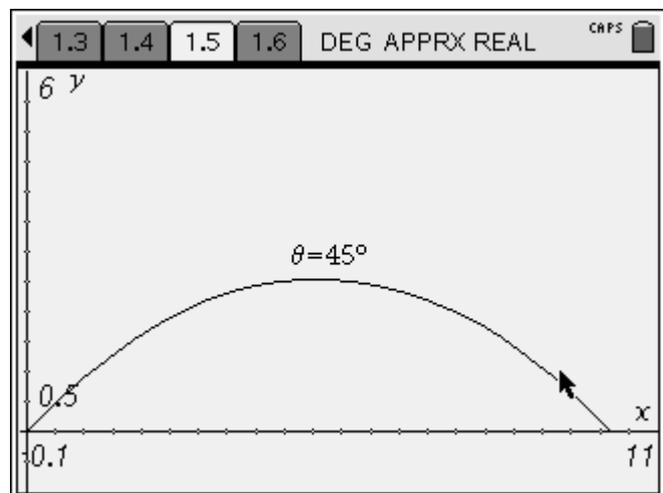
To find the maximum insert a new Calculator page, then (menu) 5: Calculus, Function Maximum.
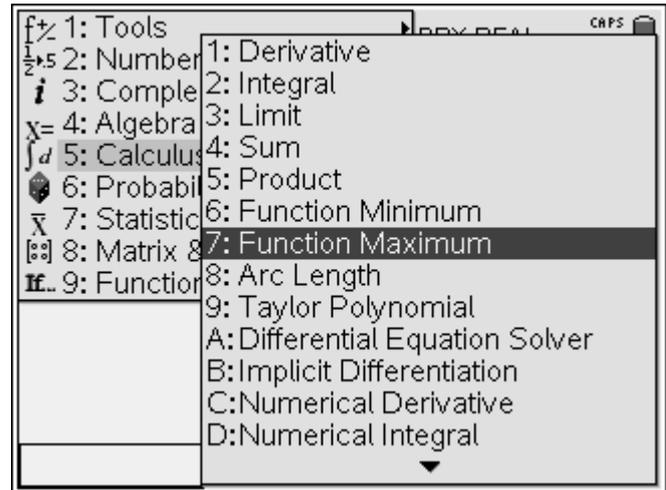
So for the angle 45 degrees the maximum occurs at (5.1, 2.55) with the furthest distance being 10.2 metres.
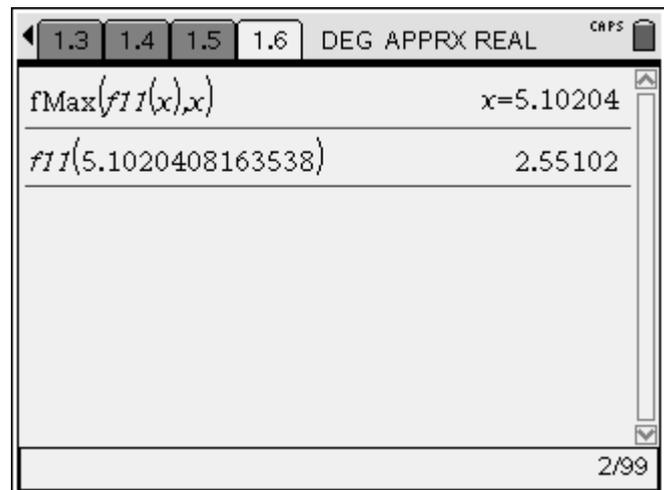
Repeat the above steps for two more selected angles e.g. 60 and 75 degrees.

So for 60 degrees the maximum occurs at (4,42, 2.51) and the function for this angle is f12.
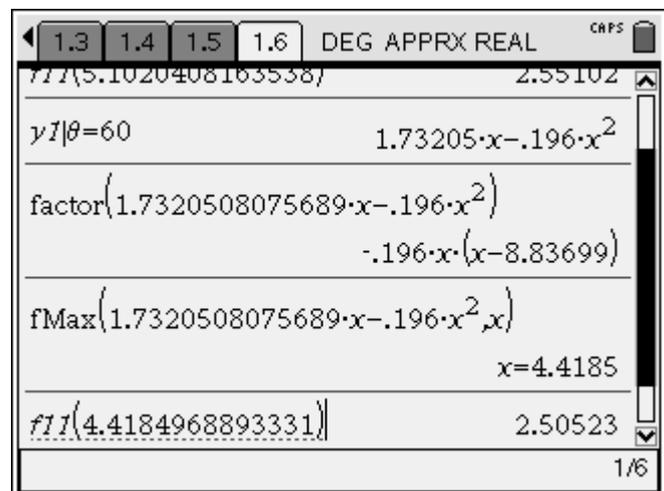
The graph shows the hose path for the three chosen angles.

The maximum for 70 degrees occurs at (2.55, 4.76).
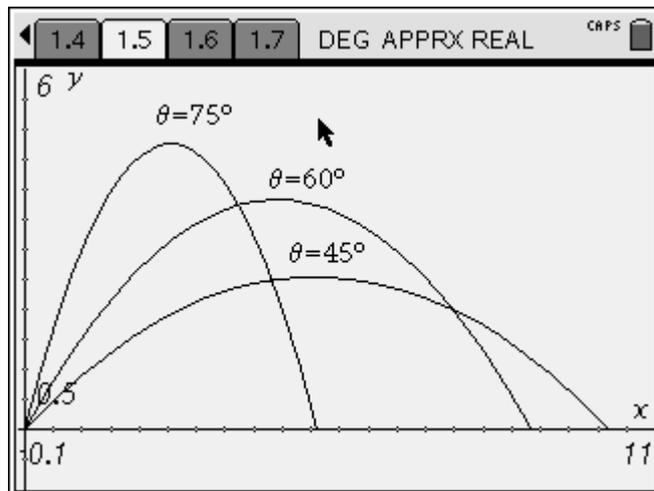
3. **i.** Explain briefly why 'equation 1' cannot be applied directly to the case where $\theta = 90°$; that is, where the nozzle is pointed vertically upwards.
   ii. By considering angles close to $90°$, or otherwise, estimate the highest point reached by a jet which goes vertically upwards. Mark this point on the axes used in **2** above.

Tangent of 90 degrees is undefined. The highest point is about 5 metres, which can be seen in diagrams below.

4. **i.** Imagine a high vertical wall at a horizontal distance of 1 metre from the nozzle. For the case where $\theta = 30°$ use your calculator to find the height at which the jet would strike the wall.
   ii. For each of the cases, which you considered in **2 ii** calculate the height at which the jet would strike the wall described in **4 i**.
   iii. For one particular value of θ, the jet will hit this wall at a higher point than for any other value. Find this height.

To find the height at which the jet would strike the vertical wall placed 1 metre from the nozzle, evaluate the values of the function in $x = 1$ as follows:
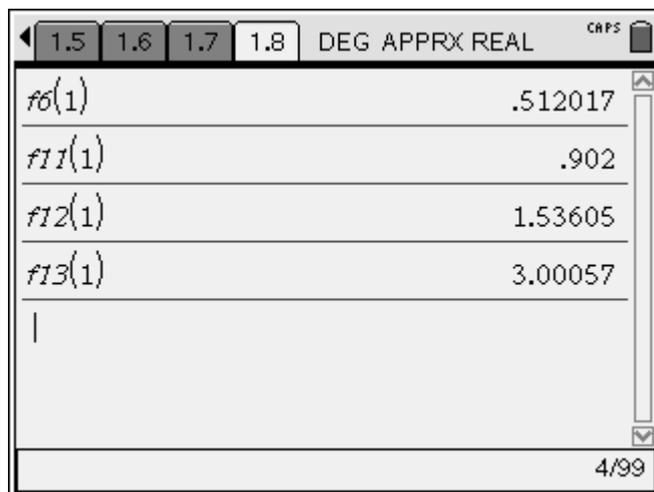
Alternatively, we may draw a wall at *x*=1 m by using (menu) 6: Points & Lines 2: Point On. Mark a point on the *x*-axis close to *x*=1. Then select (menu) 9: Construction 1: Perpendicular to draw the vertical wall.
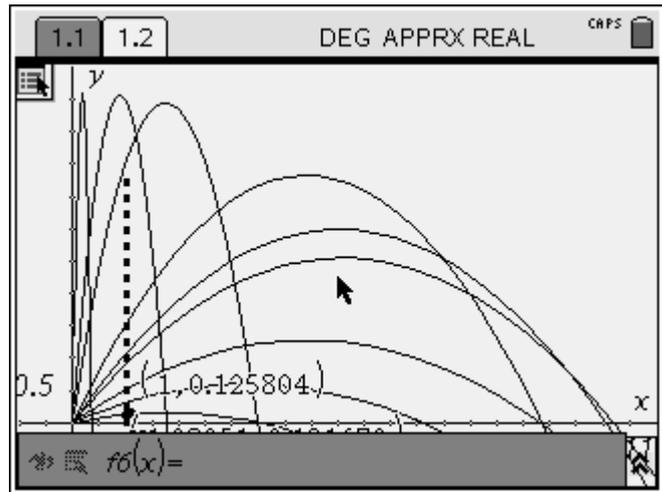
The Table may also be used to see where each jet hits the wall. (menu) 2: View: 8: Add Function Table ((ctrl)+T).
To find the angle for which the highest point on the wall can be reached we need to set the Table to Ask by selecting 5: Function Table: 3: Edit Function Table Settings, Independent: Ask, enter, then input *x = 1* and check which angle gives the highest value of *y*.
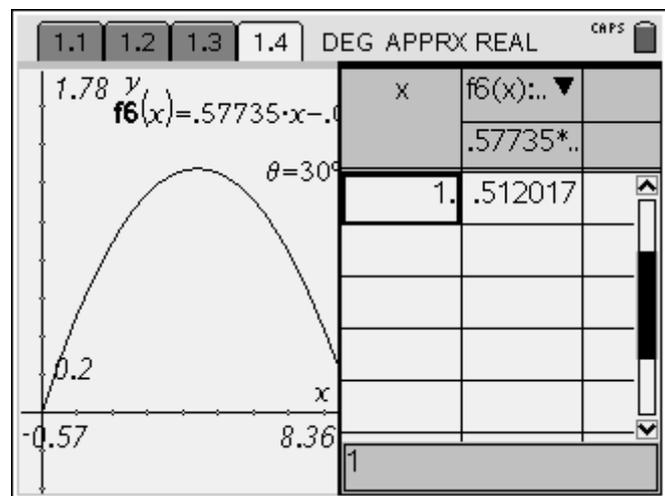
After browsing through the table it can be seen that function *f8* gives the highest *y* value of 4.98 metres, which occurs for the angle of 85 degrees.
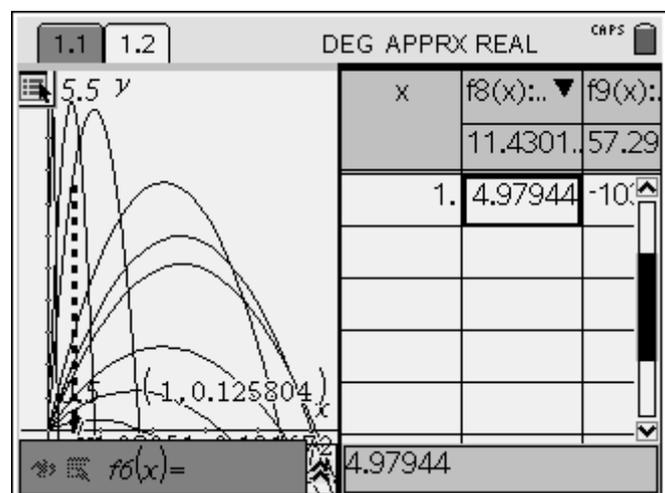
iv. Now imagine a high vertical wall at a horizontal distance of 5 metres from the nozzle. Find the highest point on this wall, which could be reached by a jet.

Repeat the above but this time set $x = 5$ m. That occurs for the angle of 60 degrees for the selected values of angles under investigation.	
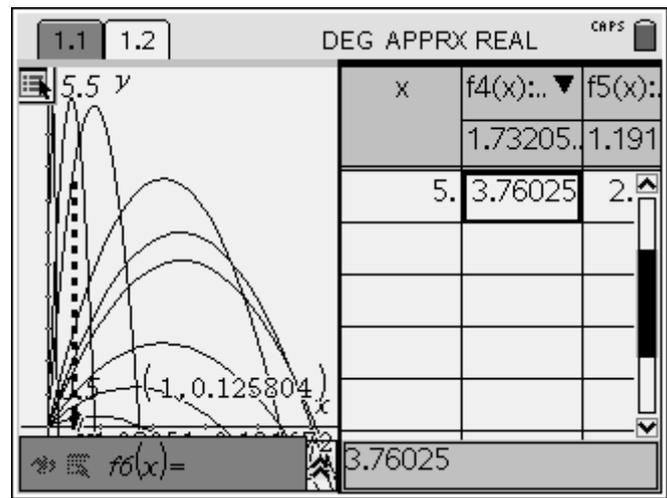
It is not the most effective way of determining the value of θ for which the jet will hit each wall at the highest point. We can set x-values appropriately to 1 and 5 and then define two new functions (also parabolic) in terms of u. Sketch the two graphs and find their maximum values to the nearest degree.

Define new functions in terms of θ first.

For the wall at 1 metre from the nozzle, the jet will hit the wall at the highest point of 5.05 metres. This will occur for the angle of 84 degrees.
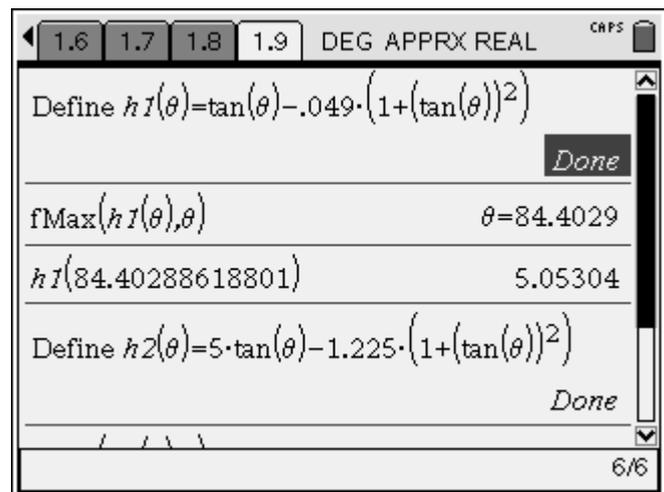
For the wall located at 5 metres from the nozzle, the jet will hit the wall at the highest point of 3.88 metres when the hose makes an angle of 64 degrees with the horizontal.
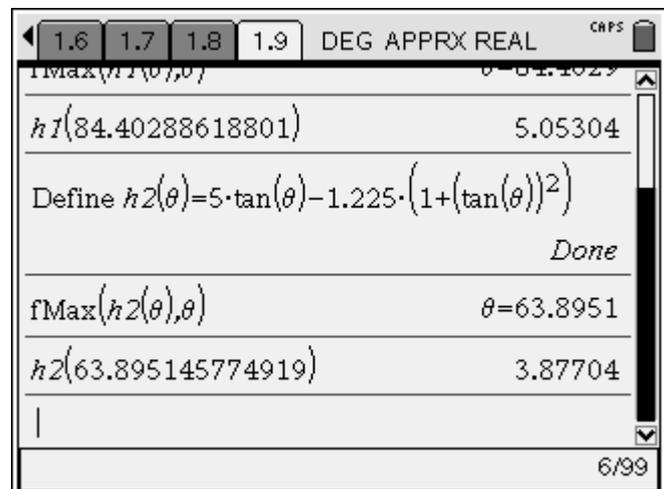
Graphically.
First define two new functions in terms of *u*.
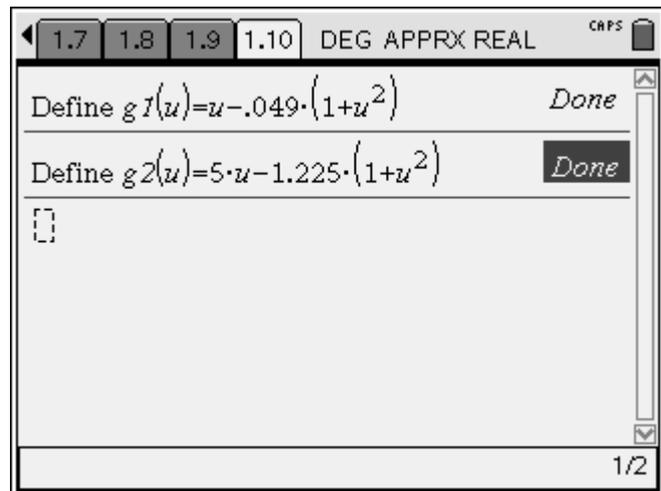
Sketch each graph separately and locate the turning point. Then calculate the respective angles.

And $\theta = \tan^{-1} 10.203 = 84^o$

Repeat for *x* = 5.
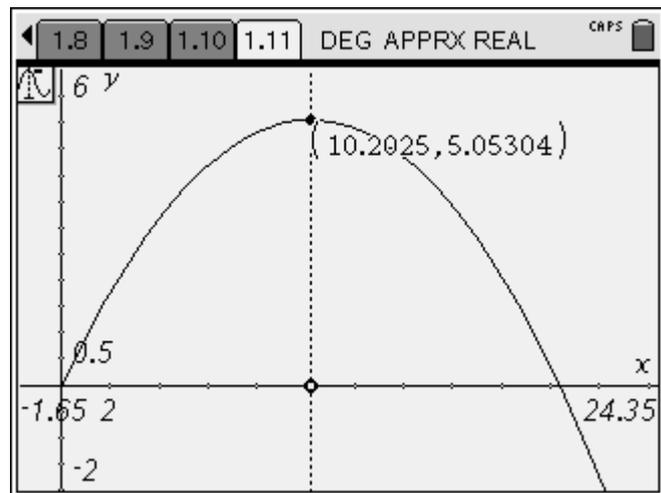
5. If the jet does **not** meet any obstacles on the way, then the furthest point on the horizontal ground that can be reached by a jet varies as the angle θ varies. Estimate the furthest point on the ground that a jet can reach and state the angle for which it occurs.

It will occur for 45 degrees. Class discussion.

6. Imagine all the jets turned on simultaneously for all angles θ such that 0° < θ ≤ 90°, like a fountain. The resulting sheet of water has a definite shape and boundary.

i.   Use your calculator to sketch about 10 graphs for the appropriate range of angles. Copy your graphs and sketch the boundary curve on the new set of axes.

ii.   Discuss how you might use your CAS calculator to find the equation of the curve of the boundary shape.

iii.   Hence find the equation of the curve.

Boundary Curve.

The following angles were chosen for the boundary curve with the corresponding functions:

θ	30	40	50	60	65	70	75	80	85	89
function	f1	f2	f3	f4	f5	f6	f7	f8	f9	f10

The new graph was drawn.
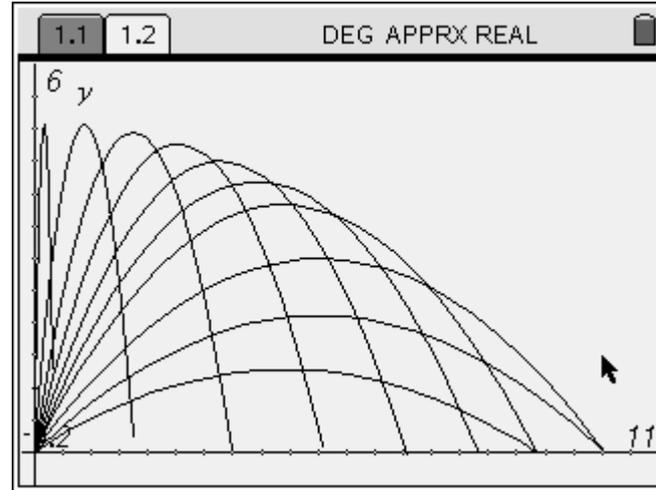
The scale can be added to this graph by selecting (menu) 2: View 5: Show Grid and then changing 1 cm to 1 m on the graph.
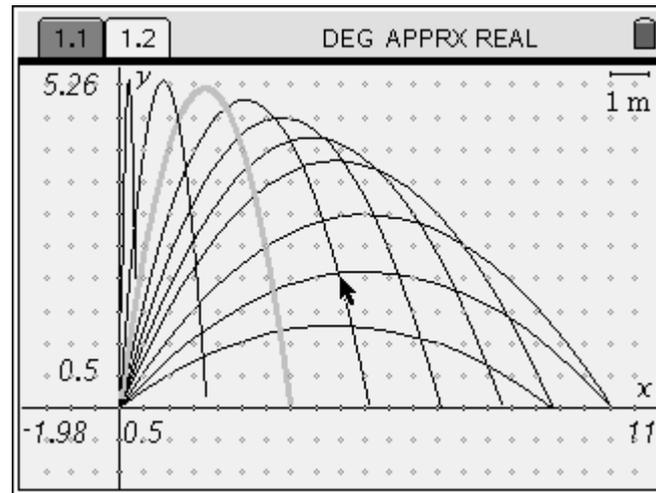
Points can be chosen manually by using (menu) 6:Points&Lines 2: Point On option and selecting points on the boundary curve and noting their coordinates.
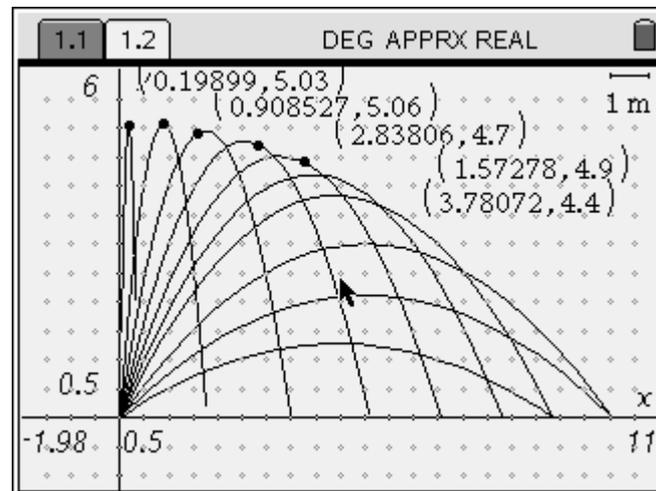
And 5 more points on the remaining part of the boundary curve.

Changing Float to 3 (8: System Info 1: Document Settings Display Digits: Float 3) will result in a smaller number of digits shown and hence in a clearer picture.
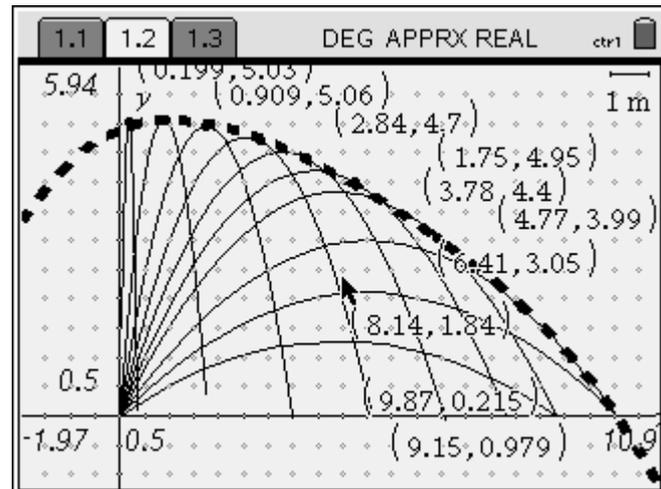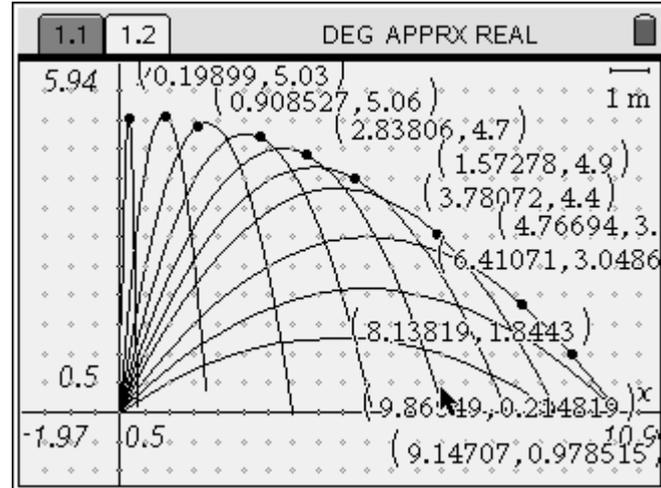
The coordinates were noted and inserted into List and Spreadsheets page. Five points were selected for a quartic regression.

The boundary curve is drawn with a dotted thick line. The equation was saved in *f13(x)*.