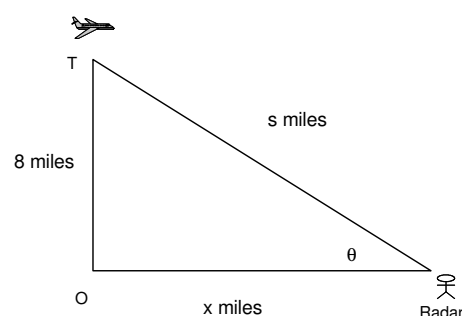


Specialist Mathematics Related Rates and Implicit Differentiation

An airplane flies directly over a radar installation at 300 mph. Its altitude is 8 miles and it is flying on level flight toward the station. Find how the rate at which the distance from the airplane to the radar station is changing and the rate at which the angle of elevation from the radar station to the plane is changing at the instant when the plane's horizontal distance to the radar station is 5 miles.



Type the expression $x^2 + 8^2 = s^2$, remembering that both x and s are functions of time t .	F1- Tools F2- R13Cbrd F3- Calc F4- Other F5- Pr3mID F6- Clean Up ■ NewProb Done ■ $(x(t))^2 + 64 = (s(t))^2$ $(x(t))^2 + 64 = (s(t))^2$ $(x(t))^2 + 64 = (s(t))^2$ MAIN DEG APPROR FUNC 2/30
Differentiate the equation implicitly with respect to t . You need to use F3 Calc 1: d(	F1- Tools F2- R13Cbrd F3- Calc F4- Other F5- Pr3mID F6- Clean Up $(x(t))^2 + 64 = (s(t))^2$ ■ $\frac{d}{dt}((x(t))^2 + 64 = (s(t))^2)$ $2 \cdot x(t) \cdot \frac{d}{dt}(x(t)) = 2 \cdot s(t) \cdot \frac{d}{dt}(s(t))$ $\frac{d((x(t))^2 + 64 = (s(t))^2)}{dt}$ MAIN DEG APPROR FUNC 3/30
Make $\frac{ds}{dt}$ the subject by dividing both sides by $2s(t)$.	F1- Tools F2- R13Cbrd F3- Calc F4- Other F5- Pr3mID F6- Clean Up $x(t) \cdot \frac{d}{dt}(x(t)) = s(t) \cdot \frac{d}{dt}(s(t))$ ■ $\frac{x(t) \cdot \frac{d}{dt}(x(t))}{s(t)} = \frac{s(t) \cdot \frac{d}{dt}(s(t))}{s(t)}$ $\frac{x(t) \cdot \frac{d}{dt}(x(t))}{s(t)} = \frac{d}{dt}(s(t))$ $\text{ans}(1)/s(t)$ Note: Domain of result may be larger MAIN DEG APPROR FUNC 4/30
Now calculate $\frac{ds}{dt}$ given that $x=5$, $\frac{dx}{dt} = -300$ and $s = \sqrt{64 + 25} = \sqrt{89}$	F1- Tools F2- R13Cbrd F3- Calc F4- Other F5- Pr3mID F6- Clean Up ■ $\frac{x(t) \cdot \frac{d}{dt}(x(t))}{s(t)} \mid x(t) = 5 \text{ and } \frac{dx}{dt} = -300$ $\frac{-1500.}{s(t)}$ ■ $\frac{-1500.}{s(t)} \mid s(t) = \sqrt{89}$ $\frac{-1500.}{\sqrt{89}} = -159.$ $\frac{-1500.}{\sqrt{89}} \mid s(t) = \sqrt{89}$ MAIN DEG APPROR FUNC 7/30
To convert the rate to km/h, type Units velocity_mph → (2ndMode)_kph The plane is getting closer to the radar station at the rate of 159 mph or 256 kph.	F1- Tools F2- R13Cbrd F3- Calc F4- Other F5- Pr3mID F6- Clean Up $\frac{-1500.}{s(t)}$ ■ $\frac{-1500.}{s(t)} \mid s(t) = \sqrt{89}$ $-159.$ ■ $-159. \text{ mph} \rightarrow \text{kph}$ -255.885 kph $-159. \text{ mph} \rightarrow \text{kph}$ MAIN DEG APPROR FUNC 8/30
To find $\frac{d\theta}{dt}$ we must enter the relationship $\tan \theta = \frac{8}{x}$	F1- Tools F2- R13Cbrd F3- Calc F4- Other F5- Pr3mID F6- Clean Up ■ $\tan(\theta(t)) = \frac{8}{x(t)}$ $\tan(\theta(t)) = \frac{8}{x(t)}$ $\tan(\theta(t)) = 8/x(t)$ MAIN RAD APPROR FUNC 1/30

and differentiate implicitly with respect to time. Then make $\frac{d\theta}{dt}$ the subject.	$\frac{d}{dt}(\cos(\theta(t))) = \frac{d}{dt}(x(t)^2)$ $-(\cos(\theta(t)))^2 = (x(t))^2$ $\frac{d}{dt}(\theta(t)) = \frac{-8 \cdot (\cos(\theta(t)))^4}{(x(t))^4}$ ans(1) * (cos(theta(t)))^2 Note: Domain of result may be larger
Use the fact that $\frac{dx}{dt} = -300$ and $\cos \theta = \frac{5}{\sqrt{89}}$ and $x = 5$	$-.32 \cdot (\cos(\theta(t)))^2 \cdot \frac{d}{dt}(x(t))$ $-.32 \cdot (\cos(\theta(t)))^2 \cdot \frac{d}{dt}(x(t))$ $96 \cdot (\cos(\theta(t)))^2$ $\frac{d(x(t), t)}{dt} = -300$ MAIN RAD APPROX FUNC 5/30
To find the angle and then $\cos^2 \theta$	$\tan^{-1}(8/5) = 1.0122$ $\cos(1.0121970114513) = .529999$ $(.5299994000321)^2 = .280899$ ans(1)^2 MAIN RAD APPROX FUNC 8/30
Finally $\frac{d\theta}{dt}$ equals 7.5748 radians per hour.	$12.673597870835 \cdot \frac{d}{dt}(\theta(t))$ 12.6736 $1 \cdot \frac{d}{dt}(\theta(t)) = 7.5748$ ans(1)/12.6736 MAIN RAD APPROX FUNC 8/30
Or converting to degrees per minute: So the angle of elevation of the plane is increasing at the rate of $7^\circ 14'$ per minute.	$1 \cdot \frac{d}{dt}(\theta(t)) = 7.5748$ $\frac{7.5748011614695}{60} = .126247$ $(.12624668602449) \rightarrow \text{DMS}$ $7^\circ 14' 24.8232''$.12624668602449 DMS MAIN RAD APPROX FUNC 10/30