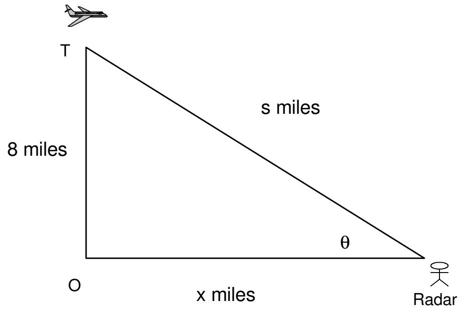
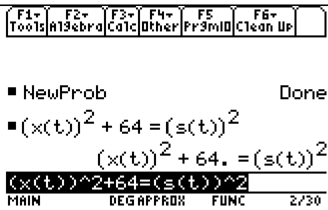
Specialist Mathematics Related Rates and Implicit Differentiation

An airplane flies directly over a radar installation at 300 mph. Its altitude is 8 miles and it is flying on level flight toward the station. Find how the rate at which the distance from the airplane to the radar station is changing and the rate at which the angle of elevation from the radar station to the plane is changing at the instant when the plane's horizontal distance to the radar station is 5 miles.



<table>
<tr><td>Type the expression $x^2+8^2=s^2$, remembering that both x and s are functions of time t.</td><td>
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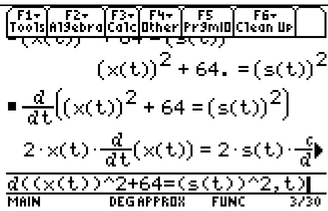
<tr><td>Differentiate the equation implicitly with respect to t.
You need to use F3 Calc 1: d(</td><td>
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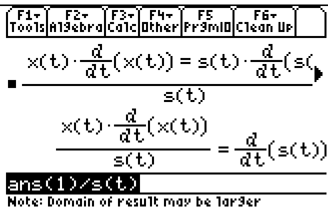
<tr><td>Make $\frac{ds}{dt}$ the subject by dividing both sides by $2s(t)$.</td><td>
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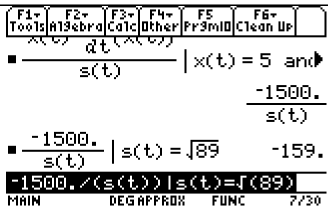
<tr><td>Now calculate $\frac{ds}{dt}$ given that x=5,
$\frac{dx}{dt}=-300$ and $s=\sqrt{64+25}=\sqrt{89}$</td><td>
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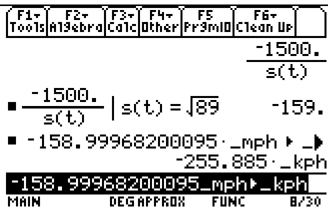
<tr><td>To convert the rate to km/h, type Units velocity_mph $\rightarrow$(2ndMode)_kph

The plane is getting closer to the radar station at the rate of 159 mph or 256 kph.</td><td>
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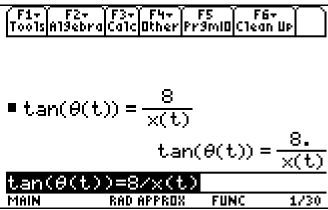
<tr><td>To find $\frac{d\theta}{dt}$ we must enter the relationship $\tan\theta=\frac{8}{x}$</td><td>
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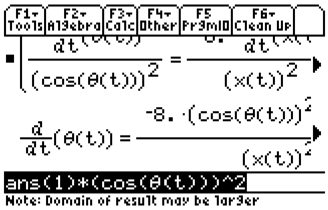
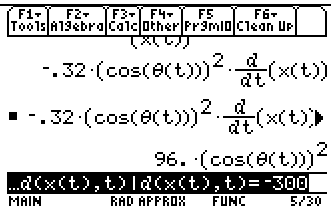
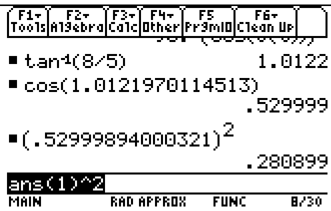
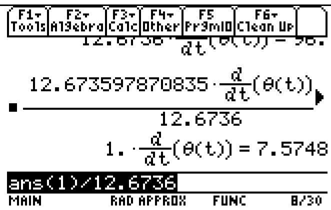
and differentiate implicitly with respect to time. Then make $\frac{d\theta}{dt}$ the subject.	
Use the fact that $\frac{dx}{dt} = -300$ and $\cos\theta = \frac{5}{\sqrt{89}}$ and $x = 5$	
To find the angle and then $\cos^2\theta$	
Finally $\frac{d\theta}{dt}$ equals 7.5748 radians per hour.	
Or converting to degrees per minute: So the angle of elevation of the plane is increasing at the rate of $7^o 14'$ per minute.