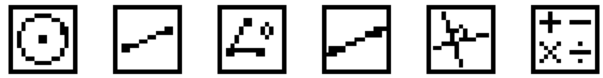
EXPLORATIONS™

Activity 27

Objective

- To investigate properties of angles and arcs formed when chords, secants, and tangents intersect and intercept arcs in a circle

Cabri® Jr. Tools



Angles Formed by Intersecting Chords, Secants, and Tangents

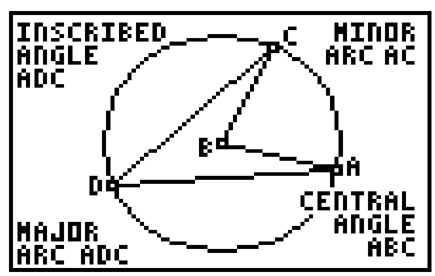
Introduction

This activity is designed to help you discover several important theorems concerning circles and arc sizes. Since the Cabri Jr. application does not have the ability to directly measure an arc angle of a circle, you will make use of the Central Angle Theorem, which ensures that the measure of a central angle of a circle is equal to the measure of its intercepted arc.

This activity uses the following definitions:

Arc — a portion of the circle whose endpoints are on the circle (arcs are measured in either degrees or radians).

Minor arc — a portion of the circumference of a circle that is less than 180° (or π radians). $\overset{\frown}{AC}$ is a minor arc.



Major arc — a portion of the circumference of a circle that is greater than 180° (or π radians). $\overset{\frown}{ADC}$ is a major arc.

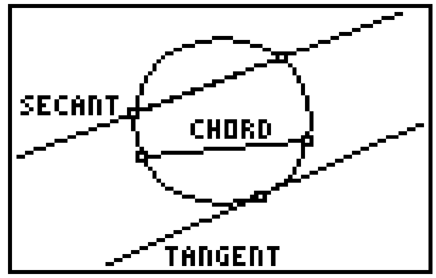
Central angle — an angle with its vertex at the center of a circle and sides that intercept an arc. $\angle ABC$ is a central angle.

Inscribed angle — an angle with its vertex on the circle and sides that intercept an arc. $\angle ADC$ is an inscribed angle.

Chord — a segment with its endpoints on the circle (the diameter is the longest chord of a circle)

Secant — a line that intersects a circle at two points

Tangent line to a circle — a line that intersects a circle at only one point called the *point of tangency*

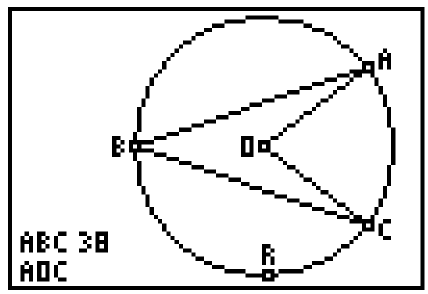


Part I: Chords Intersecting on the Circle

Construction

Construct an angle inscribed in a circle and its corresponding central angle.

- Draw a circle with a center *O* and radius point *R*.
- Construct chords $\overline{AB}$ and $\overline{BC}$ forming an inscribed angle $\angle ABC$.
- Construct the radii $\overline{AO}$ and $\overline{CO}$ forming the central angle $\angle AOC$.
- Measure $\angle ABC$ and $\angle AOC$.



Note: Not all measurements are shown.

Exploration

- Observe the relationships between $\angle ABC$ and $\angle AOC$ as you drag point *B* around the circle.
- Observe the relationship between $\angle ABC$ and $\angle AOC$ as you drag points *A* and *C*.
- Observe the relationship between $\angle ABC$ and $\angle AOC$ as you drag points *O* and *R*.

Questions and Conjectures

1. Make a conjecture about the relationship between the measure of the inscribed angle $\angle ABC$ and the measure of its intercepted arc $\overset{\frown}{AC}$ (represented by the central angle $\angle AOC$).
2. Look up theorems related to inscribed angles. Notice that when point *B* is in the interior of $\angle AOC$ or when $\overset{\frown}{AC}$ is a major arc, the results you observe do not seem to support the given theorems. Are the theorems incorrect? Explain your reasoning.
3. Using your answer to Question 1, make a conjecture about $\angle ABC$ when $\overline{AC}$ is a diameter of a circle. Explain your reasoning and be prepared to demonstrate.

Part II: Chords Intersecting Inside the Circle

Construction

Construct a circle with two intersecting chords.

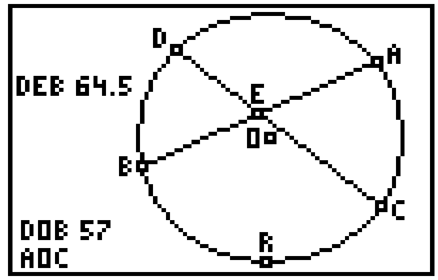
Clear the previous construction.

Draw a circle with a center *O* and radius point *R*.

Construct chords $\overline{AB}$ and $\overline{CD}$ that intersect at point *E*.

Measure $\angle DEB$ and the central angles $\angle DOB$ and $\angle AOC$.

Note: *It is not necessary to construct segments* $\overline{DO}$, $\overline{OB}$, $\overline{AO}$, *and* $\overline{OC}$ *in order to measure these angles.*



Note: *Not all measurements are shown.*

Exploration

Use the **Calculate** tool to investigate the relationship between $\angle DOB$, $\angle AOC$ (representing $\overset{\frown}{DB}$ and $\overset{\frown}{AC}$ respectively), and $\angle DEB$. Move the defining points of the chords and circle to observe the relationship for different measurements of $\angle DEB$.

Repeat the above investigation for $\angle AED$ using central angles $\angle AOD$ and $\angle COB$.

Questions and Conjectures

1. Make a conjecture about the relationship between an angle formed by chords intersecting inside a circle and the measure of the intercepted arcs. Is this relationship true for any angle formed by intersecting chords? Explain your reasoning.

2. The relationship determined in Part I can be thought of as a special case of the above relationship. Explain how that could be true.

Part III: Secants and Tangents Intersecting Outside the Circle

Construction

Construct a circle with two secants that intersect outside the circle.

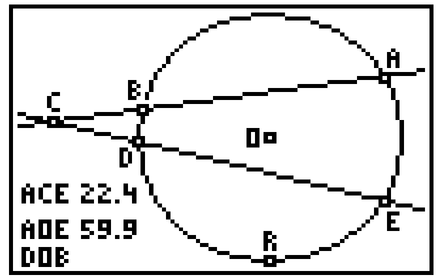
Clear the previous construction.

Draw a circle having a center *O* and radius point *R*.

Construct secant $\overleftrightarrow{AC}$ that intersects the circle at points *A* and *B*.

Construct secant $\overleftrightarrow{EC}$ that intersects the circle at points *E* and *D*.

Measure $\angle ACE$ and the central angles $\angle AOE$ and $\angle DOB$.



Note: *Not all measurements are shown.*

Exploration

Use the **Calculate** tool to investigate the relationship between $\angle ACE$ and the arcs $\overset{\frown}{DB}$ and $\overset{\frown}{AE}$. Move the defining points of the secants and circle to observe the relationship for different measurements of $\angle ACE$.

Adjust the location of points *A* and/or *C* until points *A* and *B* coincide (thus making $\overleftrightarrow{AC}$ a tangent to the circle). Observe any changes to the relationship noted above.

Adjust the location of point *E* until it coincides with point *D* (thus making both $\overleftrightarrow{AC}$ and $\overleftrightarrow{EC}$ tangent to the circle). Observe any changes to the relationship noted in the first exploration.

Questions and Conjectures

1. Make a conjecture about the relationship between an angle formed by two secants that intersect outside a circle and the measure of the intercepted arcs. Is this relationship true for any angle formed by intersecting secants? Explain your reasoning.

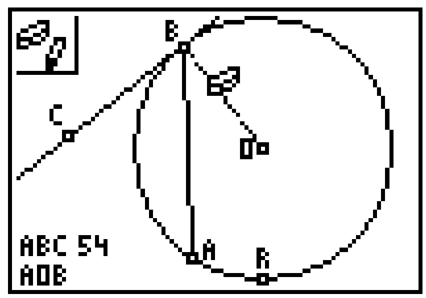
2. Make a conjecture about the relationship between an angle and the intercepted arcs when one or both of the lines that form the angle are tangents to the circle. Explain any differences between this conjecture and your answer to Question 1.

Part IV: Intersecting Tangents and Chords

Construction

Construct a circle which has a tangent and a chord that intersect at the point of tangency.

- Clear the previous construction.
- Draw a circle with a center *O* and radius point *R*.
- Construct a radius $\overline{OB}$.
- Construct a line perpendicular to radius $\overline{OB}$ passing through and defined by point *B*.
- Construct a point *C* on the line perpendicular to segment $\overline{OB}$.
- Drag point *B* around the circle to ensure that $\overleftrightarrow{BC}$ is always tangent to the circle.
- Construct a chord $\overline{AB}$.
- Measure $\angle ABC$ and the central angle $\angle AOB$.
- Hide radius $\overline{OB}$.



Note: *Not all measurements are shown.*

Exploration

Use the **Calculate** tool to investigate the relationship between $\angle ABC$ and $\overset{\frown}{AB}$. Move the defining points of the chord and the circle to observe the relationship for different measurements of $\angle ABC$.

Questions and Conjectures

1. What properties of a tangent line to a circle ensure that $\overleftrightarrow{BC}$ intersects the circle at only one point?
2. Make a conjecture about the relationship between the angle formed by the intersection of a tangent and a chord and the measure of the intercepted arc. Explain your reasoning.

Teacher Notes

Activity 27

Angles Formed by Intersecting Chords, Secants, and Tangents

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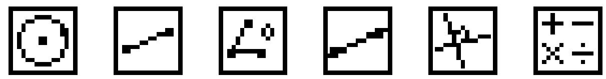
- To investigate properties of angles and arcs formed when chords, secants, and tangents intersect and intercept arcs in a circle

Cabri® Jr. Tools

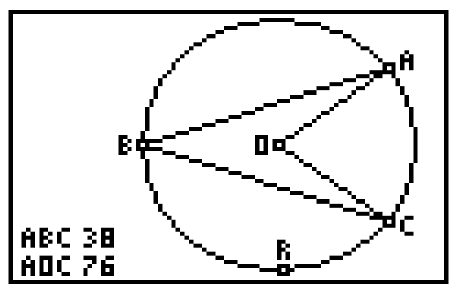


Part I: Chords Intersecting on the Circle

Answers to Questions and Conjectures

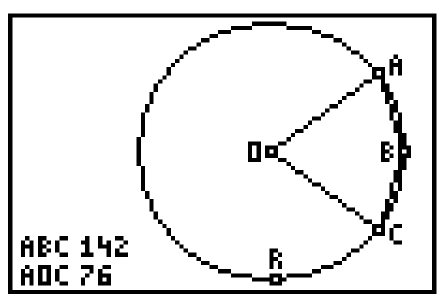
1. Make a conjecture about the relationship between the measure of the inscribed angle $\angle ABC$ and the measure of its intercepted arc $\overset{\frown}{AC}$ (represented by the central angle $\angle AOC$).

   The measure of the inscribed angle $\angle ABC$ is equal to one half of the measure of the intercepted arc $\overset{\frown}{AC}$.



2. Look up theorems related to inscribed angles. Notice that when point B is in the interior of $\angle AOC$ or when $\overset{\frown}{AC}$ is a major arc, the results you observe do not seem to support the given theorems. Are the theorems incorrect? Explain your reasoning.

   The **Angle Measurement** tool will display angle measures between 0° and 180° only. When point B is in the interior of $\angle AOC$ or when $\overset{\frown}{AC}$ is a major arc, the displayed value of the $m\angle AOC$ will be 360° - $m\overset{\frown}{AC}$. Using the actual measure of the arc will show the theorems to be correct.



3. Using your answer to Question 1, make a conjecture about $\angle ABC$ when $\overline{AC}$ is a diameter of a circle. Explain your reasoning and be prepared to demonstrate.

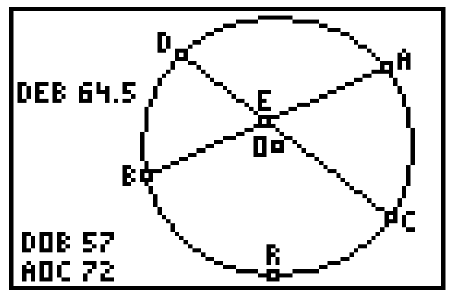
   $\angle ABC$ is a right angle since the measure of the intercepted arc is 180°.

Part II: Chords Intersecting Inside the Circle

Answers to Questions and Conjectures

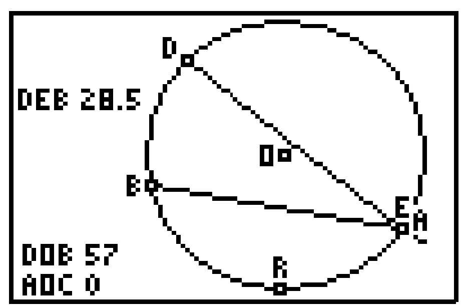
1. Make a conjecture about the relationship between an angle formed by chords intersecting inside a circle and the measure of the intercepted arcs. Is this relationship true for any angle formed by intersecting chords? Explain your reasoning.

   The measure of the angle formed by intersecting chords will equal the average of the measures of the arcs that are intercepted. For example, $m\angle DEB$ will equal the average of $m\widehat{AC}$ and $m\widehat{DB}$.



2. The relationship determined in Part I can be thought of as a special case of the above relationship. Explain how that could be true.

   This can be considered a special case of the relationship if you consider that the angle formed by the intersection of two chords will intercept two arcs on a circle. Drag point *A* until it coincides with point *C*. If the two chords actually intersect on the circle, then the measure of one intercepted arc would be zero. The average of two numbers when one of the numbers is zero is equivalent to finding half of the other number.

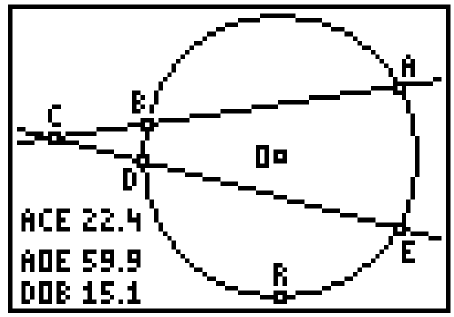


Part III: Secants and Tangents Intersecting Outside the Circle

Answers to Questions and Conjectures

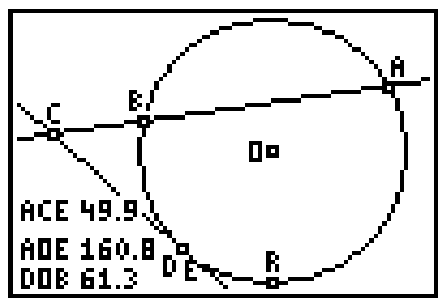
1. Make a conjecture about the relationship between an angle formed by two secants that intersect outside a circle and the measure of the intercepted arcs. Is this relationship true for any angle formed by intersecting secants? Explain your reasoning.

   The measure of the angle formed by two intersecting secants is equal to one half of the difference of the measures of the intercepted arcs.



2. Make a conjecture about the relationship between an angle and the intercepted arcs when one or both of the lines that form the angle are tangents to the circle. Explain any differences between this conjecture and your answer to Question 1.

   The same relationship as described in Question 1 holds if one or both of the lines are tangents. It may be difficult to show this for two tangents since one of the intercepted arcs will be a major arc. Since tangents to a diameter will be parallel, the **Angle Measurement** tool cannot be used directly to measure a major arc. (See Part I.)



Part IV: Intersecting Tangents and Chords

Answers to Questions and Conjectures

1. What properties of a tangent line to a circle ensure that $\overleftrightarrow{BC}$ intersects the circle at only one point?

   A line intersecting a circle at point *P* will be tangent to the circle if and only if the line is perpendicular to a radius drawn at point *P*.

2. Make a conjecture about the relationship between the angle formed by the intersection of a tangent and a chord and the measure of the intercepted arc. Explain your reasoning.

The measure of the angle formed by the intersection of a tangent and a chord will equal one half of the measure of the intercepted arc, $m\angle ABC = 1/2\ (m\ \widehat{AB})$. This relationship may be more difficult for the students to see due to the accuracy in angle measure using the Cabri® Jr. application. (See A Specific Cabri Jr. Issue on page vi.) Also, the **Angle Measurement** tool cannot be used to directly measure a major arc. (See Part I.)