Math Objectives

- Students will be able to identify situations in which blocking will reduce variation.
- Students will recognize that some randomized block designs are useful in reducing variation while others are not.
- Students will recognize that random allocation of events occurs within each block.
- Students will recognize that the experimental units *within* each block are as similar as possible and different from the experimental units in the other blocks based on a variable that has the potential to affect the outcome of an experiment.
- Students will look for and make use of structure (CCSS Mathematical Practices).
- Students will construct viable arguments & critique the reasoning of others (CCSS Mathematical Practices).

Vocabulary

- experiment
- experimental units
- random allocation
- randomized block design
- treatments

About the Lesson

- This lesson involves investigating the effectiveness of two mosquito sprays in a large tract of land by using three different experimental designs—one randomized design and two randomized block designs.
- As a result, students will:
  - Randomly allocate treatments and observe the mean number of mosquitoes in the lots after the sprays have been applied.
  - Compare mean differences and variability for the three experimental designs to determine the most effective design for this context.

TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- You can hide the function entry line by pressing ctrl G.

Lesson Files:
Student Activity
Effective_Blocking_Student.pdf
Effective_Blocking_Student.doc

TI-Nspire document
Effective_Blocking.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

TI-Nspire™ Navigator™ System

- Transfer a File.
- Use Screen Capture to examine patterns that emerge.
- Use Teacher Edition computer software to review student documents.
- Use Quick Poll to assess students' understanding.

Prerequisite activity:

- This activity assumes students have completed the Statistics Nspired Activity "Blocking Introduction."

Discussion Points and Possible Answers

Tech Tip: This activity is most effective using color software or handhelds.

Teacher Tip: Because students are generating random allocations of treatments, results will vary. Occasionally a student will get results that show a different outcome due to this randomness.

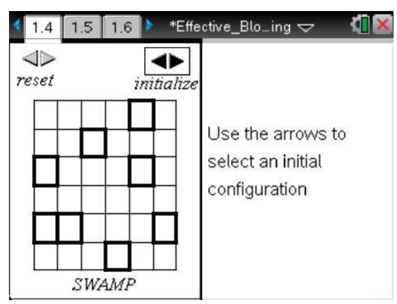
Pages 1.2 and 1.3 contain directions to "seed" your handheld for the activity.

Kate is a realtor who needs to sell 36 lots, some of which are bordered on one side by a swamp. Because of the swamp, the land is known to have a large number of mosquitoes.

Kate has researched different types of mosquito sprays in her quest to decrease the number of mosquitoes and make the land more attractive to buyers. She has permission to conduct an experiment on only eight of the lots to determine which of the top two sprays would be the most effective in reducing the number of mosquitoes. She decides to simulate some experiments to determine how she might proceed.

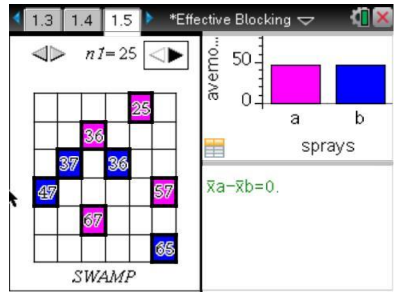
Move to page 1.4.

1. On this page, click the *initialize* arrow once to initialize the activity and select Kate's eight lots.



Teacher Tip: *Initialize* selects the plots for treatment. *Reset* wipes the field clean.

Move to page 1.5.



The first method Kate simulated was a completely randomized design. The left work area of the page displays the eight lots of land she will use for her experiment (the ones you generated on Page 1.4).

2. Now, click the right arrow above the grid one time to randomly allocate mosquito *treatments* (spray A and spray B) to the eight selected lots. Four lots will randomly be assigned to spray A (pink), and four will be randomly assigned to spray B (blue).
   - The numbers in the selected lots represent the simulated numbers of mosquitoes a week after the lots were sprayed.
   - The bar graph in the right work area shows the average number of mosquitos found after each spray was applied.
   - Beneath that is a calculation of the difference between the means of the two sprays, mean(spray A) – mean(spray B), in the number of mosquitoes remaining in each lot.

Tech Tip: Make sure students click the arrow on the right and wait a few seconds for the treatments to be assigned.

a. Does a larger or smaller number of mosquitoes indicate a more effective spray? Explain.

Sample Answers: A smaller number of mosquitoes still alive after one week indicates that the spray is more effective.

b. Note the number of live mosquitoes in each of the lots. What conjecture might seem reasonable?

Sample Answers: It looks like more mosquitoes are alive in the lots closer to the swamp, regardless of the spray.

c. Does there appear to be a difference in the mean number of live mosquitoes for the two treatments? Explain your reasoning.

> **Teacher Tip:** Students will generate different random samples based on the seeding of their calculators. Sharing their results will enable students to see that overall, they are likely to reach the same conclusion.

Sample Answers: For my trial, there does appear to be a difference in the sprays—the mean number of mosquitoes remaining with spray A is around 50 while the mean remaining after spray B is around 40. The difference between the two sprays is 15.25 mosquitoes, suggesting spray B is better.

d. How would you interpret $\bar{x}a - \bar{x}b < 0$?

Sample Answers: On average, there are more mosquitos in the lots that were sprayed with spray B than in the lots sprayed with spray A.

3. a. Click the right arrow above the grid again to see another simulation of a possible random treatment allocation. How does this new sample change your answers to questions 1b and 1c above?

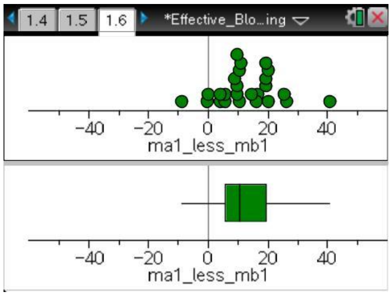
Sample Answers: The numbers of live mosquitoes in the lots were different. Spray B still killed more mosquitoes, but the difference was smaller this time, 4.75. And again, more mosquitoes were alive closer to the swamp for both sprays.

b. Click the arrow to create about 23 more simulations, each time observing the number of live mosquitoes in the lots on the grid and in the bar graph. What are the noticeable patterns in the number of mosquitoes? Explain your reasoning.

> **Tech Tip:** Press [tab] until the arrow you want is selected. Press [enter] to activate the slider. Now you can use the arrow button on your handheld instead of clicking.

Sample Answers: There are two patterns in the data. First, Spray B seems to be generally more effective at killing mosquitoes. Second, consistently more mosquitoes seem to be alive closer to the swamp.

Move to page 1.6.



4. The plot on Page 1.6 represents the differences (mean Spray A – mean Spray B) in the number of mosquitoes for each random allocation of treatments you generated on Page 1.5.
   a. What does the vertical line in the plot represent in the context of the problem?

Sample Answers: The vertical line represents a mean difference of zero in the number of mosquitoes killed by the two sprays (spray A minus spray B).

   b. What does a positive value in the dotplot of accumulated data indicate in the context of mosquitoes? What does a negative value indicate in the context of mosquitoes?

Sample Answers: Since the points represent the (mean for spray A – mean for spray B), a positive value indicates that there are more mosquitoes after spray A was used, and therefore, spray B was more effective. A negative value indicates that there are more mosquitoes after spray B was used, and therefore, spray A was more effective.

   c. What is the meaning of the difference between 0 and the mean? Explain your answer.

Sample Answers: A difference between 0 and the mean indicates how many more mosquitoes the more effective spray killed.

   d. Describe the variability in the mean differences using randomized sampling.

Sample Answers: Although in two cases, spray A was better than spray B (the mean difference was negative), half of the simulations had a mean difference of 4.75 to 18.5 between the two sprays.

   e. Would you consider one mosquito spray to be more effective than the other at reducing the number of mosquitoes?

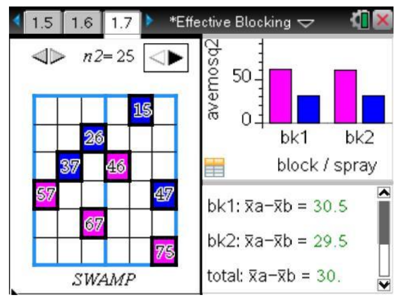
Sample Answers: Spray B appears to be more effective. The boxplot of mean differences has a median of about ten mosquitoes remaining, suggesting that spray B killed on average about ten more mosquitoes than spray A.

Teacher Tip: While most students will conclude that spray B is more effective, because the lots are randomly selected and the sprays are randomly assigned to the lots, it is possible for a student to show no difference or that spray A is more effective.

f. In addition to the type of spray used, name another factor that might affect the number of mosquitoes present on Kate's land.

Sample Answers: In all of the samples drawn, more mosquitoes were left alive closer to the swamp—regardless of which spray was used.

Move to page 1.7.



Pooja has an idea for Kate. Since many mosquitoes seem to be near the swamp, she wants to divide the lots vertically down the middle so that each block would have an equal number of lots close to the swamp. Then she will randomly assign the sprays within the two blocks.

5. On this page, click the right arrow in the left panel to randomly assign the treatments, spray A and spray B, to the lots within each of the blocks in Pooja's design.
   a. Inspect the bar graphs. Does there appear to be a difference in the mean number of live mosquitoes for the two treatments? Explain your reasoning.

Sample Answers: For my trial, the mean number of mosquitoes left after Spray A is larger than the mean after Spray B in both blocks.

b. Interpret each of the three expressions in the lower right corner of your screen.

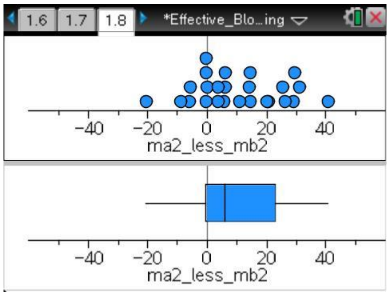
Sample Answers:

bk1: $\bar{x}a - \bar{x}b$	The mean number of mosquitoes in the two lots using spray A in block 1 minus the mean number of mosquitoes in the two lots using spray B in block 1.
bk2: $\bar{x}a - \bar{x}b$	The mean number of mosquitoes in the two lots using spray A in block 2 minus the mean number of mosquitoes in the two lots using spray B in block 2.
total: $\bar{x}a - \bar{x}b$	The mean number of mosquitoes using spray A minus the mean number of mosquitoes using spray B.

c. Click the arrow 24 more times, each time observing the number of live mosquitoes in the lots on the left and the bar graph on the right. What observations can you make about the number of live mosquitoes for each spray?

Sample Answers: Sometimes the difference in the mean number of mosquitoes remaining was larger for Spray A in one block and Spray B in the other; sometimes it was the other way around. Sometimes Spray A had a larger mean for both blocks and sometimes the other way around. It was hard to see any pattern.

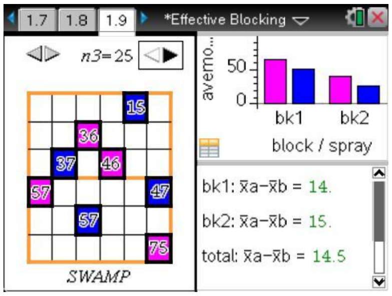
Move to page 1.8.



6. The plot on this page represents the mean differences, mean(Spray A) – mean(Spray B), in the number of mosquitoes from the simulations you generated on Page 1.7. Would you consider one mosquito spray to be more effective at reducing the number of mosquitoes than the other? Explain your reasoning.

Sample Answers: The lower quartile is at 0.25, so three-fourths of the differences are positive, indicating that Spray A is leaving more mosquitoes alive than Spray B.

Move to page 1.9.



Sean wonders whether it would work to divide the land horizontally at the middle so that the lots closest to the swamp are in the same block. He then randomly assigns the sprays within the two horizontal blocks.

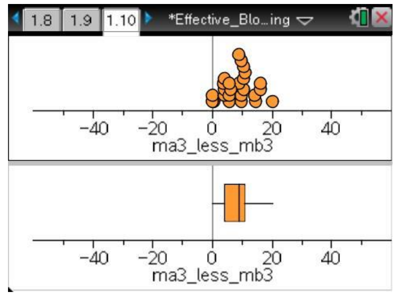
7. On this page, click the right arrow above the grid to randomly assign the treatments, spray A and spray B, to the lots within each of these two blocks. The bar graph on the right shows the difference in the number of mosquitoes within each block that are still alive.
   a. What do you observe from the bar graphs?

Sample Answers: More mosquitoes are alive in block 1, which is probably because it is the block closest to the swamp.

b. Click the arrow 24 more times, each time observing the number of live mosquitoes in the lots on the left and the bar graph on the right. What observations can you make about the number of live mosquitoes for each spray?

Sample Answers: Pretty consistently the mean number of mosquitoes left alive was larger for Block 1. Usually spray A had a larger mean for both blocks but not always—fewer mosquitoes were killed.

Move to page 1.10.



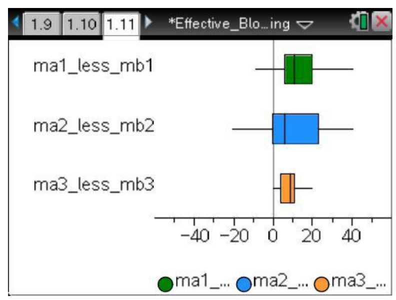
8. The plot on this page represents the mean differences, mean(Spray A) – mean(Spray B), in the number of mosquitoes from the simulations you generated on Page 1.9.
   a. What is the median of the distribution, and how would you describe it in the context of the problem?

<u>Sample Answers:</u> The median is about 10, so in about half of the simulations the difference between the two means, mosquitoes left by spray A minus mosquitoes left by spray B, was more than 10.

   b. Is your conclusion from the horizontal blocks different from your conclusion based on the vertical blocks? Explain.

<u>Sample Answers:</u> The median was positive in both the boxplots of the vertical and horizontal blocks so both indicated that spray A left more mosquitoes alive than spray B—so spray B is more effective.

Move to page 1.11.



9. This page shows boxplots of the mean differences in the numbers of living mosquitoes from the designs that Kate, Pooja, and Sean suggested—completely randomized design, vertical blocks, and horizontal blocks.
   a. Compare the medians of the boxplots of the three experimental designs.

<u>Sample Answers:</u> The medians were very close; two of them were 9.75 and the third was 10.75.

   b. Compare the variation in the boxplots of the three experimental designs.

<u>Sample Answers:</u> The smallest range and interquartile range was for the horizontal blocks, with an IQR of 10.5 (from 4.75 to 15.25) and a range of 27 (from -1 to 26). The IQR for the vertical blocks was 25 (from 0.25 to 25.25), and the range was 40 (from -10.25 to 30.25). The range for the random design is the same as the range for the vertical blocks; the IQR is smaller than for the random design but not as large as for the vertical blocks.

TI-Nspire Navigator Opportunity: *Screen Capture*
See Note 1 at the end of this lesson.

10. Spray B kills on average ten more mosquitoes per lot than spray A. Based on your simulations for the three methods, discuss the likelihood of making an incorrect decision about which spray is more effective?

Sample Answers: For both the SRS and the vertical blocks, approximately 25% of the samples had a negative difference, incorrectly indicating that spray B was more effective. Only one sample using the horizontal blocks would have made that incorrect decision.

11. a. The goal of the experiment was to determine which mosquito repellent is more effective at reducing the number of mosquitoes on Kate's land. Kate will only get to run her experiment one time. Which method, completely randomized, horizontal blocking or vertical blocking, should she chose for the experiment, and why?

Sample Answers: Student responses should take into consideration the center value and the spread. Based on the median values, all of the strategies suggested that spray B was better than spray A, but the horizontal blocking reduced the variation more than either of the other two, so this is what I would choose. Smaller variation means that there is less of a risk of making the wrong decision.

b. When Sean studied blocking, he learned that the experimental units within each block should be as alike as possible. How do the results on Page 1.11 support this requirement?

Sample Answers: The horizontal blocks reduced the variability more than the other two designs. The lots in one block were all close to the swamp and lots in the other block were all farther away from the swamp. This seems to support the idea that the experimental units (lots) within a block should be as alike as possible.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 2 at the end of this lesson.

Wrap Up

Upon completion of the lesson, the teacher should ensure that students are able to understand:

- Some situations contain factors that make it necessary to use a randomized block design.
- Some randomized block designs are useful in reducing variation while others are not.
- Random allocation of treatments occurs within each block.
- The experimental units *within* each block are as similar as possible and different from the experimental units in the other blocks based on a variable that has the potential to affect the outcome of an experiment.
- The experimenter often only gets one chance to run the experiment and collect the data needed to make a decision regarding the effectiveness of the treatments. An appropriate experimental design that has minimum variation should be used.

Assessment

Identify each as always, sometimes or never true. Be ready to justify your reasoning.

1. Blocking reduces variation.
   Answer: Sometimes
2. The experimental units within a block should be randomly assigned treatments.
   Answer: Always
3. One blocking design will be more effective at reducing variation than another.
   Answer: Sometimes
4. Each block should be as different from other blocks as possible.
   Answer: Always

TI-Nspire Navigator

Note 1

Question 9, Name of Feature: Screen Capture

Use TI-Nspire™ Navigator™ to see if the answers to 8b are common—that the smallest range and IQR are typical for the horizontal blocks. Students will get different data because of the random allocation of treatments. Discussion of these differences and the randomness behind them is important.

Note 2

Question 11, Name of Feature: Quick Poll

A Quick Poll can be given at the conclusion of the lesson. You can save the results and show a Class Analysis at the start of the next class to discuss possible misunderstandings students might have.