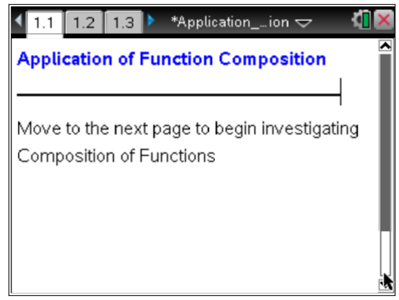
Application of Function Composition
Student Activity

Open the TI-Nspire document *Application_of_Function_Composition.tns.*

This activity will explore the composition of functions, their graphs, and their application to real-world situations. Throughout the lesson pay particular attention to the order of operations of the function compositions.



Move to page 1.2.

Press ctrl ▶ and ctrl ◀ to navigate through the lesson.

1. The composition of two functions is when you evaluate one function in terms of another function whereby the range of the first function determines the domain of the second function. This page shows the graphs of $f(x) = x^2$, $g(x) = x - 3$, $(f \circ g)(x) = f(g(x)) = (x-3)^2$. The function $(f \circ g)(x)$ is the composition of the functions $f(x)$ and $g(x)$. Describe the graph of the functions $f(x)$, $g(x)$, and $(f \circ g)(x)$ by answering the following questions.

    a. How does the composition function $f(g(x))$ affect the graph of $f(x)$ and $g(x)$? Explain.

    b. Using the graphs generated, determine the domain and range of $f(x)$, $g(x)$, and $f(g(x))$. Explain your reasoning.

    c. From the graph, what are the domain and range of $(g \circ f)(x)$? How does this differ from your answer in part a? Explain your reasoning.

    d. What type of graph will the function $(g \circ f)(x)$ produce? Explain your reasoning.

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2. This page shows the graph of $(g \circ f)(x)$.

    a. Explain how the composition between f and g translates the graphs of each function into the new function $(g \circ f)(x)$ and its graph.

    b. Why is the graph not the same as $(f \circ g)(x)$? Explain your reasoning.

3. Now consider the functions $k(x) = x - 4$ and $m(x) = \sqrt{x}$. Find $m(k(x))$ algebraically. What is the domain and range of $m(k(x))$? Explain.

Move to page 1.4.

4. Graph functions $k(x) = x - 4$, $m(x) = \sqrt{x}$, and $m(k(x))$ on the graphs page by moving the cursor to the entry line at the bottom of the screen. You may graph these functions as $f5(x)$, $f6(x)$, and $f7(x)$. Explain how the graphs of m(x), $k(x)$, and $m(k(x))$ relate to the algebraic solution for finding the domain and range you found in question 3.

5. You have already discovered that the relationship between the composition of two functions is that the range of the inside function determines the domain of the outside function.

   There are many real-life applications of composition of functions that occur. One such application is change in the area of a circular oil spill. Oil is spilling from a well at a rate of 2,500 gallons per day. At this rate the radius of the circular pool of oil will change at a constant rate of 4 miles per day.

   a. Write an expression that represents the radius of the oil slick at time *t*, *r* days after the leak has started.

   b. What are the domain and the range of the radius? Explain your reasoning.

   c. Write an expression to represent the area in terms of the radius and explain how the relationship between the radius and the area of the pool represent the composition of the two functions.

   d. What are the domain and range of the expression you created to represent the area of the oil spill? Explain your reasoning.

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6. Page 1.5 contains an animation of an oil spill. By pressing **Play** on the animation, data will be collected in a spreadsheet on page 1.6. Describe mathematically how the animation reflects the situation of the oil spill.

Move to page 1.6.

7. Page 1.6 contains a spreadsheet that represents the data collected from the model of oil spilling from an oil well in a large body of water. In this case, the area of the circular spill is a composition of the functions of change in radius and elapsed time. The data you collected from step 6 is the data that should appear in the spreadsheet.

   a. Explain how the data collected in the spreadsheet represents the functions you defined in questions 5a and 5b.

   b. What will be the shape of the graph of the function for **r**(*t*)? How do you know? Explain your reasoning.

   c. What will be the shape of the graph of the function for *A*(*r*)? How do you know? Explain your reasoning.

Move to page 1.7.

8. Page 1.7 contains an empty graph. Move the cursor on the handheld to the bottom of the screen where it says “click to add the variable” and select the variable that represents the independent variable. Press tab and click to add the dependent variable on the vertical axis. This should represent the radius as a function of time. Did your conjecture from question 7b hold true for the shape of the graph? Explain why or why not.

Move to page 1.8.

9. Page 1.8 contains an empty graph. Repeat the steps in question 8. Move the cursor on the handheld and click to add the variable that represents the independent and dependent variables for the area as a function of the radius. Did your conjecture from question 7c hold true for the shape of the graph? Explain why or why not.

Move to page 1.9.

10. Part of the discovery of composition of functions involves realizing that the range of the inside function, in this example $r(t)$, becomes the domain for the outside function, $A(r(t))$. What do you think the graph of $A(t)$ should look like? Explain.

    On this page, move the cursor on the handheld and click to add the variable that represents the independent variable, time, and dependent variable, area, for the area as a function of time. What do you notice about the shape of the graph when compared to the shape of the graph when area was a function of the radius? Explain.

Move to page 1.10.

11. To further explore the concept of how the area of the circular pool of oil is a function of the radius, which in turn is a function of time, and that the area is the composition of the functions of time and radius, use the handheld to move the open circle that represents time. Since you have discovered that $r(t) = 4t$ and that $A(r) = \pi r^2$, find a function that models the mathematical relationship of the area in terms of time.