

## Activity 3

### To Infinity and Beyond!

#### Objectives

- Develop an understanding of what it means to take a limit “at” infinity
- Develop an understanding of behavior that prevents limits from occurring by means of chaos or oscillation
- Estimate limits from graphs and tables of values
- Connect the ideas of end behavior, horizontal asymptotes, and limits at infinity

#### Materials

- TI-84 Plus / TI-83 Plus

#### Introduction

The limit of a function is the behavior the output value approaches as the input value approaches a particular value. Limits are also examined when the output value keeps increasing or decreasing without bound. In this case the notation is

$$\lim_{x \rightarrow \infty}$$

read “the limit as the input value approaches infinity.” The limit exists when the function moves toward a single output value and fails to exist when it does not move toward one value.

This activity has three parts. First, you will examine, graphically and numerically, the behavior of functions as the input approaches infinity. Next, you will examine graphically limits that do not exist because of continued chaotic output behavior as the input values continue to approach a particular value. Finally, you will examine a variety of limit problems.

#### Exploration

1. Input this function into **Y1** in the **Y=** editor:

$$f(x) = \frac{2x^2 + 200x + 1000}{x^2 + 1}$$

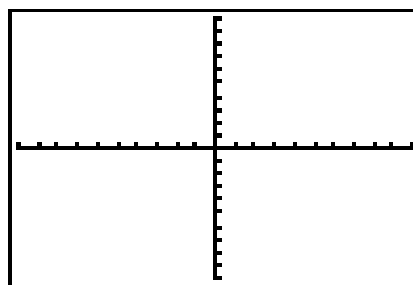
2. Record the function values for the inputs {1, 2, 3}.

3. Store the output values in a list as shown.

Y1({1,2,3})→L1

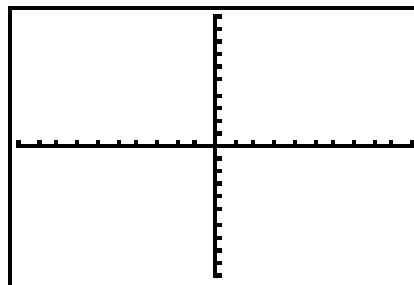
L1	L2	L3	1
60	-----	-----	
281.6			
161.8			
-----			
L1(1)=601			

4. Repeat the process for the input values {100, 200, 300}. Store the output values in **L2**, and record the values.
5. Repeat the process once more for the input values {1,000, 2,000, 3,000}. Store the output values in **L3**, and record the values.
6. Look at the values in all three lists, and draw a conclusion regarding the behavior of the function.
7. As the input value increases without bound, you can say that  $x$  is *approaching* infinity. To confirm this conclusion, try the input values  $\{10^{10}, 10^{15}, 10^{20}\}$ , and record the results.
8. Does it appear that the function actually reaches the exact value of the limit? Is this a reasonable result? Explain.
9. Look at the behavior graphically. Start with the standard viewing window, and sketch the graph.



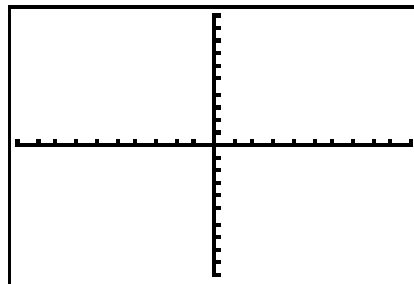
- 10.** In what viewing window would it be reasonable to see the behavior from the input values listed in Question 2? Complete the window at the right, and then sketch the graph.

```
WINDOW
Xmin=
Xmax=
Xscl=
Ymin=
Ymax=
Yscl=
Xres=
```



- 11.** In what viewing window would it be reasonable to see the behavior from the input values listed in Question 4? Complete the window at the right, and then sketch the graph.

```
WINDOW
Xmin=
Xmax=
Xscl=
Ymin=
Ymax=
Yscl=
Xres=
```

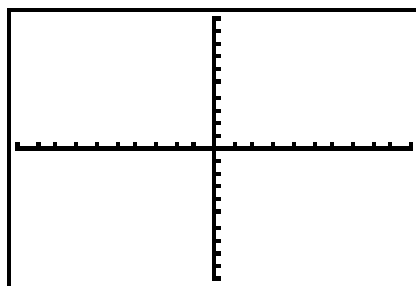


12. In what viewing window would it be reasonable for you to see the behavior from the input values listed in Question 5? Complete the window at the right, and then sketch the graph.

```

WINDOW
Xmin=
Xmax=
Xscl=
Ymin=
Ymax=
Yscl=
Xres=

```



13. Estimate

$$\lim_{x \rightarrow \infty} \frac{2x^2 + 200x + 1000}{x^2 + 1}$$

14. Investigate and draw a conclusion of

$$\lim_{x \rightarrow -\infty} \frac{2x^2 + 200x + 1000}{x^2 + 1}$$

15. Investigate

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x$$

- In **L1**, record the values for  $x = \{1, 2, 3\}$
- In **L2**, record the values for  $x = \{100, 200, 300\}$
- In **L3**, record the values for  $x = \{1,000, 2,000, 3,000\}$ .

X	L1	X	L2	X	L3
1		100		1000	
2		200		2000	
3		300		3000	

16. Record the values for  $x = \{10^{10}, 10^{15}, 10^{20}\}$ .

What do you notice?

17. Graph the function in the viewing window shown, and then sketch the graph. Use **TRACE** to help you describe what you see.

```

WINDOW
Xmin=-1*10^8
Xmax=1*10^12
Xscl=0
Ymin=0
Ymax=4
Yscl=1
Xres=1
  
```



18. State and explain

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x$$

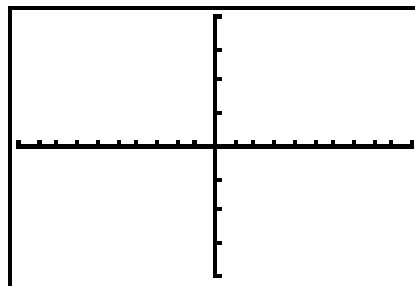
19. Examine the behavior of the function

$$f(x) = \sin\left(\frac{1}{x}\right)$$

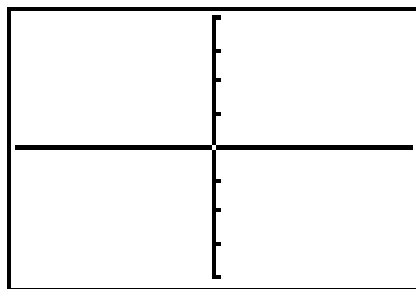
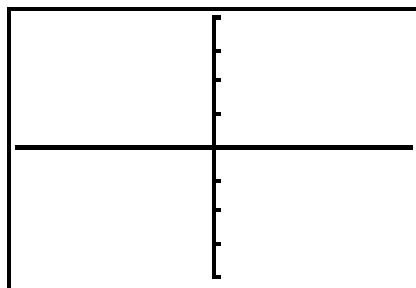
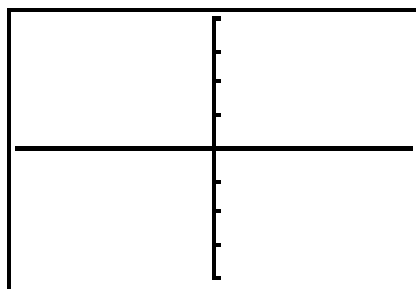
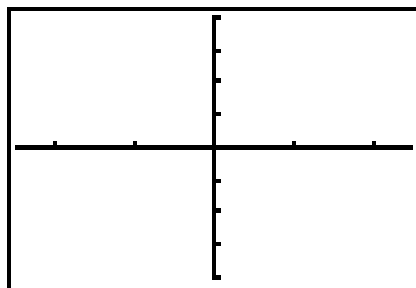
as the input approaches 0 graphically by using the viewing window shown. Make sure that your graphing handheld is in **Radian** mode. Sketch the graph.

```

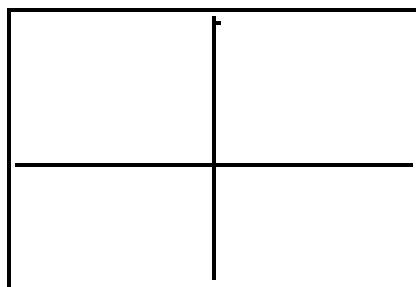
WINDOW
Xmin=-10
Xmax=10
Xscl=1
Ymin=-4
Ymax=4
Yscl=1
Xres=1
  
```



20. Set the **ZOOM FACTORS** to **XFact** = 4 and **YFact** = 1. Zoom in at  $x = 0$  four consecutive times. Sketch what you see for each zoom.



21. Create a small zoom box around the origin, and sketch the graph you see.



22. What can you conclude about  $\lim_{x \rightarrow 0} \sin\left(\frac{1}{x}\right)$ ?

Find the limit for each expression.

23.  $\lim_{x \rightarrow 0} \frac{\sin(x)}{x}$

24.  $\lim_{x \rightarrow \infty} \frac{x-2}{2x+5}$

25.  $\lim_{x \rightarrow 0} \frac{|x|}{x}$

26.  $\lim_{x \rightarrow 0} \frac{\sin(5x)}{4x}$

27.  $\lim_{x \rightarrow -\infty} \frac{4x-1}{\sqrt{x^2+2}}$

28.  $\lim_{x \rightarrow 1} \frac{x^3-1}{x-1}$

29.  $\lim_{x \rightarrow 2} \left( \frac{1}{x-2} - \frac{4}{x^2-4} \right)$

