Using the Document

PWL_Definite_Integral_Function.tns:

On page 1.2, a function f is presented as a piecewise defined linear graph. The vertices that connect the linear pieces of the graph can be moved up or down (in integer steps) by grabbing any marked point on the graph and dragging to another location. The values of x and a can also be changed by grabbing the corresponding point and dragging along the horizontal axis. These values can also be manipulated by using the sliders in the left pane. For a fixed value a, the value of $g(x)$ is displayed in the bottom pane.

On page 2.2, a function f is presented as a piecewise defined linear graph in the top pane. The vertices can be moved up or down, and the values of x and a can be changed on the graph or by using the sliders. The graph of the function g is displayed in the bottom pane.

Suggested Applications and Extensions

Page 1.2

Use the default function f to answer Questions 1-8. Remember that g is a function of x (for a fixed value of a). The values of a and x can be manipulated, the value $g(x)$ is displayed in the bottom pane, and the shaded region in the top pane represents the accumulated net area bounded by the graph of f and the horizontal axis from a to x.

1. Find the domain and range of the function g.
2. Use the graph to explain geometrically how to find $g(0), g(1), g(2)$, and $g(3)$.
3. On what intervals is g increasing? Decreasing?
4. Find $g(-4)$. Explain this answer since the shaded region representing $g(-4)$ is above the horizontal axis.
5. On the interval $[-4,4]$ where does g have an absolute maximum value? Explain the behavior of the function f at and around this value.
6. On the interval $[-4,4]$ where does g have an absolute mimimum value? Does this contradict the Extreme Value Theorem? Why or why not?
7. Let $a = -4$. Explain how the values in Question 1 change.
8. Let $a = 0$. Explain how the values in Question 1 change.
9. Let $a = -4$. Move the points to construct a piecewise defined linear function such that the maximum value of g occurs at $x = 0$. Let $a = -2$. Where does the absolute maximum occur now?
10. Let $a = 0$. Move the points to construct a non-zero piecewise defined linear function such that $g(4) = 0$. Let $x = 4$. Explain the relationship among the shaded regions in the graph of f to the left of $x = 0$.

11. Let $a = 0$. Move the points to construct a non-zero piecewise defined linear function such that the function g is increasing on the interval $[-4, 4]$. In words, describe any special characteristic of your function f. Does this suggest another relationship between g and f? If so, explain.
12. Let $a = -4$. Is it possible to construct a non-zero piecewise defined linear function such that $g(4) = 25$? If not, why not? If so, construct one such function.

Page 2.2

Use the default function f to answer Questions 1-9. Remember that g is a function of x (for a fixed value of a). The values of a and x can be manipulated, the value $g(x)$ is displayed in the left pane, and the shaded region in the top pane represents the accumulated net area bounded by the graph of f and the horizontal axis from a to x. The bottom pane displays a complete graph of the function g.

1. Find the domain and range of g.
2. Find the value $g(-1)$. Explain how to determine this value geometrically.
3. Find the value $g(0)$. Explain this answer geometrically.
4. On what intervals is g increasing? Decreasing? What are the values of f on each of these intervals?
5. For what values of x does g have a relative minimum value? Relative maximum value?
6. Where does g attain its absolute maximum value? Absolute minimum value?
7. On what intervals is g concave up? Concave down? Explain the behavior of the function f on each of these intervals.
8. Find any points of inflection on the graph of g. Explain the behavior of the graph of f at each corresponding x-coordinate.
9. Use your answers to questions 1-8 to suggest a relationship between g and f.
10. Let $a = 0$. Move the points to construct a non-zero piecewise defined linear function such that the function g is increasing on the interval $[-4, 4]$. In words, describe any special characteristic of your function f.
11. Let $a = 0$. Move the points to construct a non-zero piecewise defined linear function such that the graph of the function g has two relative maximum points and two relative minimum points.
12. Let $a = 0$. Is it possible to construct a non-zero piecewise defined function such that the graph of the function g has three relative maximum points? If not, why not? If so, then construct one such graph.
13. Let $a = -2$. Move the points to construct a non-zero piecewise defined linear function such that the graph of the function g is concave down over its entire domain.
14. For your graph constructed in Question 13, explain what happens to the graph of g as a changes.
15. Let $a = 0$. Move the points to construct a non-zero piecewise defined linear even function. Is the function g even, odd, or neither?
16. Let $a = 0$. Move the points to construct a non-zero piecewise defined linear odd function. Is the function g even, odd, or neither?