NUMB3RS Activity: All Shook Up
Episode: “Democracy”

Topic: An elementary introduction to inductive reasoning **Grade Level:** 9 - 10
Objective: Learn how inductive reasoning can solve a problem about handshakes.
Time: 10 - 15 minutes

Introduction

In “Democracy,” Charlie tries to convince baseball statistics expert Oswald Kitner (who originally appeared in the *NUMB3RS* episode “Hardball”) to join CalSci’s academic program by giving him the following problem:

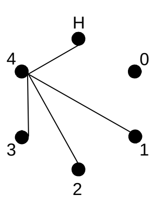
> Suppose there are five couples at a party. People shake hands, but no one shakes hands with the person they came with. At one point, one man asks the nine others how many hands they shook, and gets nine different answers. How many hands did the man himself shake?

This is not the classic problem of counting the number of possible handshakes (see “Extensions”). This activity introduces the notion of people in couples and provides a guided exploration of a way to solve the problem Charlie poses. In this activity, students will use inductive reasoning to solve this problem.

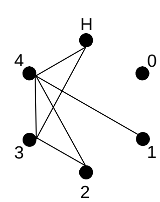
Discuss with Students

This activity is designed to provide an introduction to the use of inductive reasoning, and is intended for a wide range of student ages and abilities. The type of inductive reasoning used in this activity should not be confused with formal mathematical induction. In this particular problem, the student actually *deduces* a specific piece of the problem, and then *inductively* uses the same reasoning to determine subsequent pieces, until the complete solution is determined. A vertex-edge graph is a key tool in this induction.

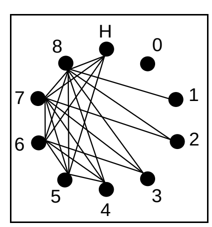
Student Page Answers:
1. *4* **2.** *0, 1, 2, 3, 4* **3.** *P4; P0 and P4 must be a couple because P4 did not shake hands with P0.*
4. *P3; P1 and P3 must be a couple.*
5. *The vertex labeled 1 (for P1).*



6. *From the graph, it is clear that vertex 2 (for P2) is also finished. Being the only one left makes it the partner of the host. The host shook hands with the two people who shook 3 and 4 hands, who are P3 and P4, respectively.*



7. *Following the same reasoning, the 9 people shook from 0 to 8 hands. The person who shook 0 hands must be the partner of the one who shook 8. Removing these two, the person who shook 1 hand shook with no one else in the remaining couples and is the only one who did not shake hands with person 7, making them partners. Continuing inductively, person 2 is the partner of person 6, and person 3 goes with person 5. The only person left is the one who shook 4 hands, who must be the partner of the host. From the graph, the host shook 4 hands.*



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Name: ______________________________ Date: ______________

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Start with a slightly easier problem. Suppose there are only six people in three couples.

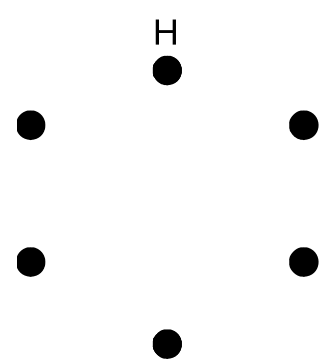
1. If no one shakes hands with his or her partner (or with himself or herself), what is the largest number of hands that anyone could shake?

2. One person (the host) asks the other five people how many hands they shook, and he gets five different answers. Make a list of the responses the host receives.

3. Use the list from Question 2 to name each person. For example, call the person who shook hands with no one P0. From the answers to Questions 1 and 2, someone shook hands with everyone except one person. Who is it? What must be true about this person and the person who did not shake anyone's hand?

4. Remove the two people identified in Question 3 from the list. Notice that P1 is still on the list, and did not shake hands with anyone else who is still on the list (because P1 shook hands with P4). Meanwhile, another person still on the list shook hands with everyone possible who is still on the list. Who is it? In the same manner as the answer to Question 3, what does this say about these two people?

5. The complete solution to this easier problem can be seen using a vertex-edge graph. The host is labeled H. Label each of the other vertices as P0 through P4, indicating the number of hands each person shook. Start with the vertex with the largest number and draw an edge (line) to each person he shook hands with. Note how this shows the answer to Question 3. Circle this vertex and this person's partner to show that they have finished shaking hands. Which other vertex has also finished shaking hands?



6. Draw the edges for the handshakes of the highest remaining number. Note how this shows the answer to Question 4. Use the graph to identify the last pair of partners and determine how many hands the host shook. Whose hands did the host shake?

7. Use similar reasoning to solve the problem Charlie posed to Oswald Kitner.

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The goal of this activity is to give your students a short and simple snapshot into a very extensive mathematical topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

Introduction

Many students may already be familiar with the classic counting problem called the “handshake problem,” usually stated as, “If there are *n* people in a room and they all shake hands with one another, how many handshakes will there be?” There are several ways to approach this problem.

For the Teacher

The general expression for the number of handshakes in the classic handshake problem is $\frac{n(n-1)}{2}$. Time and opportunity permitting, exploring how to produce this answer makes an excellent activity. One approach is to suppose that each person shakes hands with as many people as possible. Person 1 can shake $n - 1$ hands; person 2 can only shake $n - 2$ (he has already shaken person 1’s hand), person 3 shakes $n - 3$, etc. The total number of handshakes is $(n - 1) + (n - 2) + \cdots + 1 + 0$ (the last person is 0 because his or her hand has already been shaken by everyone else). For students familiar with arithmetic series, the sum is $\frac{n}{2}(n-1) = \frac{n(n-1)}{2}$.

Another way is to consider that each of the *n* people shakes $n - 1$ hands, for a total of $n(n - 1)$ handshakes. However, this expression counts every handshake twice, since when someone shakes another person’s hand, the other person is shaking back. Therefore, the final answer is $\frac{n(n-1)}{2}$.

A third (more combinatorial) approach is to reword the problem to ask how many pairs (handshakes) there are in a group of *n* people. The answer is ${}_nC_2$, or $\frac{n!}{(n-2)!2!} = \frac{n(n-1)}{2}$.

For the Student

Here are a few fun problems to think about based on handshakes:

1. If everyone in your math class shook hands with everyone else, how many handshakes would there be? How long would it take?

2. Come up with a reasonable answer to “How long would it take for everyone in the world to shake hands with everyone else?” Explain any assumptions you make.

3. How many different ways can two people shake hands if each person can shake with either hand or even both hands?

4. Answer Question 3 for beings who have 3 hands, 4 hands, or *h* hands.

There is an excellent “real-world” application of this problem. Each day that the Supreme Court meets, there is a tradition that all justices shake hands with one another to show that they have a common purpose, even if they differ in opinion (Chief Justice Fuller, who served from 1888 to 1910, started this tradition). A detailed lesson plan is available from Illuminations at:
http://illuminations.nctm.org/LessonDetail.aspx?ID=L630

For an applet for the handshake problem, see:
http://illuminations.nctm.org/ActivityDetail.aspx?ID=126

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