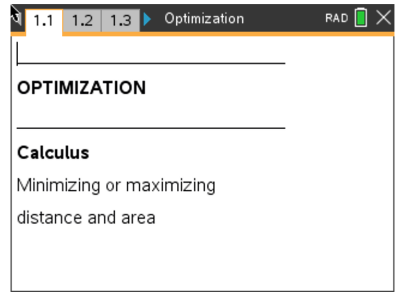
Optimization
Student Activity

Name ________________
Class ________________

In this activity, students will learn how to use the second derivative test to find maxima and minima in word problems and solve optimization in functions and parametric functions. Students will be finding a function's critical points by hand and through the handheld.



Open the file *Optimization.tns* to help guide you through this activity.

Move to **page 1.2**.

Problem 1 – Optimization of distance and area

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On page 1.3, graph the line $y = 4x + 7$. Place a point on the line and then construct a segment from the point to the origin. Discuss with a classmate how you would find the length of the segment and the coordinates of the point.

1. Explain what point you think minimizes the distance from the point to the origin.

2. State the function you are trying to minimize.

3. State the constraint.

4. Write the function to minimize using one variable.

On page 1.8, find the exact coordinates that minimize the distance using the **Derivative** and **Solve** commands. To do this, find the first derivative, solve to find the critical value(s), and then find the second derivative to confirm a minimum.

5. Find the x- and y-coordinates of the point.

6. Find the minimum distance.

Your goal in this next part is to find the dimensions of a rectangle with perimeter 200 meters whose area is as large as possible.

On page 1.12, construct a rectangle and use the **Length** tool to find the perimeter. Adjust the size of the rectangle until the perimeter is 200 m. Then, use the **Attributes** tool to lock the measurement of the perimeter.

7. State the dimensions that you think maximize the area.

8. State the function you are trying to maximize.

9. State the constraint.

10. Write the function to maximize using one variable.

Find the dimensions that maximize the area using the **Derivative** and **Solve** commands.

11. Find the dimensions of the rectangle.

Move to **page 2.1**.

Problem 2 – Optimization of time derivative problems (HL only)

A boat leaves a dock at 1 pm and travels north at a speed of 20 km/h. Another boat has been heading west at 15 km/h. It reaches the same dock at 2 pm. Your goal is to find the time when the boats were closest together. Use *t* for time.

12. Find the position function for the boat heading north.

13. Find the position function for the boat heading west.

14. State the function you are trying to minimize.

15. State the constraints.

16. Write the function to minimize using one variable.

17. State why there is a domain restriction.

Find the time at which the distance between the two boats is minimized using the **Derivative** and **Solve** commands.

18. Find the minimum distance.

19. Find the time at which this occurs. Remember to convert the value of *t* to minutes.

Extension – Parametric function (HL only)

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A projectile is fired with the following parametric functions:

$x = 500\cos(30°)t$, $y = 500\sin(30°)t - 4.9t^2$

20. Find the time when the projectile hits the ground.

21. Find how far the projectile travels horizontally.

22. Find the maximum height that the projectile achieves.

<u>Further IB Application</u>

A company selling frozen concentrated orange juice needs to manufacture a can that will save them money. This cylinder shaped can will be modelled after the image below where *r* represents the radius of the circular base and *h* represents the height of the can.

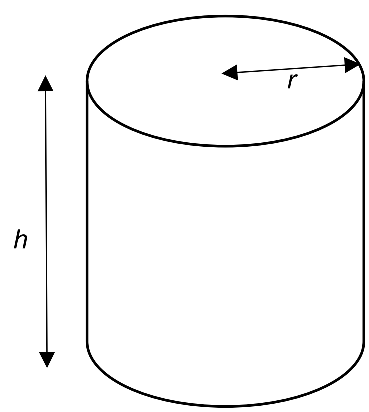

Diagram not to scale.

The sum of the radius and height for this cylinder needs to be 20 cm. The company is trying to maximize the area of the curved surface.

(a) Find an equation for the area of the curved surface in terms of the radius (*r*).

(b) Find any critical points of the equation you found in part a. Verify if these critical points are local minimums or maximums.

(c) Find the maximum area of the curved surface.